Monopsony with Recruiting

Justin Bloesch[*] Birthe Larsen[†] Anders Yding[‡]

October 14, 2025

Abstract

We develop a model of wage posting and on-the-job search where firms use wages and recruiting expenditures to attract workers. We capture three sources of labor market monopsony power: preference heterogeneity, search frictions, and labor market concentration. The model allows firms' labor supply curves to be perfectly elastic in the long run but inelastic in the short run, consistent with evidence that firms pay higher wages while growing even though the wage premium at large firms is small. We provide empirical evidence that labor supply curves are perfectly elastic in the long run using the effect of export demand shocks on the wage growth of job switchers in Denmark. Our results imply that monopsony rents are dissipated by recruiting costs, which can reconcile existing estimates of monopsony power with the profit share of national income in rich countries.

[*]Department of Economics and School of Industrial and Labor Relations, Cornell University. (email: jb2722@cornell.edu)

[†]Department of Economics, Copenhagen Business School (email: bl.eco@cbs.dk)

[‡]Department of Economics, University of California, Berkeley (email: ayding@berkeley.edu)

We thank Anastasia Burya, Gabriel Chodorow-Reich, Ryan Chahrour, Kyle Herkenhoff, Lawrence Katz, Philipp Kircher, Patrick Kline, Jeremy Lise, Alan Manning, Simon Mongey, Chris Moser, Suresh Naidu, Kris Nimark, Ezra Oberfield, Frank Pinter, Tommaso Porzio, Seth Sanders, Anna Stansbury, Evan Soltas, Jacob Weber, and seminar participants at the Copenhagen Business School, the University of California, Berkeley, the IAB workshop Imperfect Competition in the Labor Market, the NBER SI, the EALE conference, Bergen, the EEA conference, Rotterdam, the local-MPC workshop, Copenhagen, the BSE forum on the Macroeconomics of the Labor Market, Barcelona, the SED conference, Barcelona, and Columbia Junior Micro/Macro Labor conference and for helpful comments. We thank Henry Tan, Michael Yin, and Nathan Mascia for excellent research assistance. JEL codes J42, J63, E25. Keywords: Monopsony, on-the-job search, recruiting costs, labor share.

1 Introduction

There is a growing consensus that firms have significant power to set wages: firms that choose lower wages do not lose all of their employees (Card, 2022). In many standard models of labor market monopsony, firm wage setting power implies a finite labor supply elasticity, allowing researchers to infer the markdowns of wages below firms' marginal product of labor, and by extension, the profits that firms earn by exploiting their wage setting power. Yet, questions remain in estimating the labor supply elasticity and its implications for the distribution of income. First, common estimates of labor supply elasticities between 2 and 6 imply that firms must pay significantly higher wages to become large. In tension with this evidence, large firms pay only slightly higher wages in cross-sectional data, and other studies find labor supply elasticities that are at least an order of magnitude higher.[1] Additionally, numerous recent studies show that *growing* firms pay a wage premium.[2] Second, there is a puzzle regarding profits: commonly used low labor supply elasticities imply implausibly large profit shares of income in the aggregate.[3] Further, many monopsony models abstract from recruiting activity and assume that the only way firms can expand is by offering higher wages. In this article, we therefore ask: *if firms can spend on recruiting, what is the long-run labor supply elasticity they face, and how large are the profits that result from their wage-setting power?*

To answer these questions, we develop a tractable on-the-job search model in which firms attract workers on two margins: higher wages and recruiting expenditure. Workers have time-varying, horizontally differentiated preferences over firms, and the presence of both search frictions and these preferences gives firms wage-setting power: firms that choose lower wages do not immediately lose all their workers. Firms can recruit new workers by spending on recruiting activity, and the number of matches a firm generates in a period increases in both its recruiting expenditure and its stock of incumbent workers. *In the short run* the firm faces diminishing returns from its recruiting expenditure, and so in response to a positive demand shock for its output, the firm offers higher wages to increase the number of hires from a limited pool of workers matched to the firm. Consequently, firms pay high wages

[1] Sokolova and Sorensen (2021) provide a meta-analysis of two broad approaches to estimating labor supply elasticities that provide dramatically different estimates. Bloom et al. (2018) show that the firm-size wage premium had declined dramatically in the United States. Matsudaira (2014) finds that nursing homes subject to employment level mandates did not increase wages.

[2] See Schmieder (2023), Tanaka et al. (2023), and Carrillo-Tudela et al. (2023). Engbom et al. (2023) find that conditional on capital and productivity, large firms do not pay higher wages.

[3] Quoting Manning (2021), pg 11: "The low estimated wage elasticity of the labor supply curve implies that employers have a lot of monopsony power: If this power is exercised it is not clear how it can be reconciled with observed levels of the profit share."

when they are growing. However, *in the long run*, a greater number of incumbents in the firm eases recruiting, which reduces the incentive to pay high wages to generate more hires. If the firm's recruiting function has constant returns to scale, then the optimal wage in steady state is the same regardless of the firm's size, resulting in an infinite long-run labor supply elasticity. We then show that in steady state, the *recruiting cost-adjusted* wage markdown is tightly related to this long-run labor supply elasticity: if firms' long-run labor supply curve is elastic, then the recruiting cost-adjusted wage markdown is equal to one, and the gap between wages and the marginal product is entirely consumed by recruiting expenditure. Thus with constant returns to scale in recruiting, monopsony rents are dissipated by the costs incurred to acquire workers.[4]

Next, we estimate the parameters of the recruiting cost function and the long-run inverse labor supply elasticity using the effect of idiosyncratic export product demand shocks on the paths of firm employment levels, firm hiring rates, and wages. In particular, we study the wage growth for job switchers around firm-level export demand shocks using Danish administrative matched employer-employee and trade data (Hummels et al., 2014). We focus on the wage growth of switchers, rather than the wage response of stayers, because the response of stayer wages to firm demand shocks may reflect rent sharing motivations that are unrelated to the firm's long-run labor supply elasticity (Kline et al., 2019; Garin and Silvério, 2024; Carvalho et al., 2024). We find that a firm's employment level, hiring rate, and the wage growth of the firm's newly hired workers increase on impact to export demand shocks. This is consistent with diminishing returns to recruiting effort, as firms resort to offering high wages to increase hiring among a limited pool of potential workers. We also show that the effect of the demand shock on a firm's employment level is persistent: firms are larger for years following the export demand shock. Crucially, however, workers who switch into the shocked firm in the years following the trade shock, when the firm is larger but is no longer rapidly hiring, do not see any additional wage growth relative to workers who were hired prior to the shock. This is consistent with a labor supply curve that is elastic in the long run: the firm has increased in size but pays similar wages as when the firm was smaller. Together, these results are consistent with diminishing returns to recruiting expenditure in the short run, but an elastic labor supply curve in the long run.

To evaluate the extent of monopsony profits in a setting with labor market concentration, we then extend the model to consider non-atomistic firms and strategic interactions in the presence of a recruiting margin, using our estimates of the recruiting cost parameters. Firms compete according to an extended version of Bertrand competition, where firms solve their

[4] This result that a constant returns to scale recruiting function makes the firm's labor supply curve elastic is a familiar result from Manning (2003), Kuhn (2004), Manning (2006).

problems taking their competitors' wage and vacancy choices as given. We show increasing labor market concentration such that the labor market Herfindahl-Hirschman Index (HHI) increases from 0 to an empirically realistic level of 0.1 (Berger et al., 2022) lowers wages relative to marginal product by a modest 2.5 percent economy-wide. Because markdowns are not very wide, the output losses from markdown dispersions are similarly modest: Reallocating labor to equalize marginal products would increase output by only 0.6 percent.

Lastly, we demonstrate the profit puzzle: in a model with standard labor supply elasticities but no recruiting margin, and otherwise standard values for price markups and the elasticity of output with respect to capital, the profit share of income is 10–15 percentage points too high, and the labor share of income is 10–15 percentage points too low, relative to national accounts data in rich countries. We show that if firms have a recruiting function that exhibits constant returns to scale and face an elastic long-run labor supply curve, then the puzzle can be reconciled: Our model matches aggregate profit and labor shares while maintaining realistic price markups over marginal cost. Due to the modest effects of labor market concentration on the labor share, our model can still match the labor share in a setting with both recruiting costs and labor market concentration.

Our finding of elastic firm-specific labor supply curves and small profits from monopsony power contrasts with three separate strands of the monopsony literature that use different methods to estimate labor supply elasticities, all finding elasticities in the range of 2–6. Simple models dynamic monopsony based on Manning (2003) infer labor supply elasticities from the elasticities of recruiting and separation rates to wages but assume no recruiting margin. Our model nests these simple models and allows for an elastic labor supply curve due to firms' ability to spend on recruiting, taking as given this literature's empirical estimates of recruiting and separation elasticities. Relative to papers that estimate labor supply curves using the effect of demand shocks on firm size and wages (such as Lamadon et al. (2022), Seegmiller (2021) and Chan et al. (2023)), we find similar employment and wage responses of stayers to shocks, but we find more elastic labor supply curves when we use the wages of new hires, as suggested by our model. Finally, relative to models with Cournot competition in the labor market and no recruiting margin (such as Berger et al. (2022)), our model with Bertrand competition and recruiting results in higher labor supply elasticities, more compressed markdowns, and less misallocation from markdowns dispersion for a given level of labor market concentration. On the whole, our results still point to a labor market with a substantial rents: Our evidence is consistent with wages that are around 8% below marginal product and a marginal hiring cost of a worker that corresponds to 4–6 months of wages. However, our results suggest that competition among firms, via recruiting expenditure, limits the extent to which these labor-market rents translate into aggregate profits.

One theoretical contribution we make is that our environment has random on-the-job search and firm wage setting, but there is a point mass of wages in equilibrium when firms and workers are ex-ante homogeneous, in contrast to Burdett and Mortensen (1998). Like Albrecht et al. (2018), we achieve this by workers having horizontally differentiated preferences over workplaces, but unlike much of the existing monopsony literature, we make these differentiated preferences time-varying. Our paper shares with Heise and Porzio (2022) the combination of firm wage setting, on-the-job search, and idiosyncratic non-wage preferences.

Our findings have implications for the labor search literature. In many structural labor search models, firms have linear production technologies, and firm size is pinned down by a convex recruiting cost function (e.g. Acemoglu and Hawkins (2014)). Instead, we present evidence for a recruiting cost function that is homogeneous of degree one where firm size affects matching rates (Coles and Mortensen, 2016; Burdett and Vishwanath, 1988; Gouin-Bonenfant, 2022; Kuhn, 2004), so that the size of multi-worker firms is pinned down by diminishing returns in either production or product demand. This distinction is substantive, as our model and empirical findings show that the former set of assumptions implies wide markdowns (inclusive of recruiting costs) that generate the counterfactually low labor share that motivates this paper.

The paper is organized as follows. Section 2 lays out the model of dynamic monopsony with atomistic firms and decomposes marginal product into wages, recruiting costs, and labor market profits. Log-linearizing the firm's problem, we show that new hire wages are a function of current and future hiring rates and employment levels. Section 3 presents the empirical evidence on the wage growth of switchers around firm export demand shocks in Denmark and estimates the key parameters in the recruiting cost function. Section 4 extends the model to include labor market concentration and addresses the question of aggregate profit and labor shares. Section 5 concludes.

2 Monopsony with Recruiting: Atomistic Firms

In this section, we derive our model of monopsony with recruiting, where atomistic firms set wages and choose how much to spend on recruiting. We define the equilibrium and characterize firms' steady-state wage and employment policy. We decompose the share of marginal product that is spent on wages, spent on recruiting, and retained by firms as profits. We show that this decomposition depends on (i) the sensitivity of recruiting and separation with respect to firm wage policies, and (ii) the parameters of firms' recruiting cost function. In particular, if firms have a constant returns to scale recruiting function, then firms face a perfectly elastic labor supply curve in the long run, and all of the marginal product is exhausted by wages and recruiting costs. We show that the parameters of the firm's recruiting-cost function—and, by extension, the firm's long-run labor-supply elasticity—can be estimated from the paths of firms' new-hire wages, hiring rates, and employment levels in response to idiosyncratic product demand shocks. Section 3 describes the empirical implementation.

2.1 Setup

Time, Agents, and Local Labor Markets Time is discrete, and the model will later be calibrated to monthly frequency. There is a unit mass of both workers i and firms j. Firms are atomistic in the sense that they have zero mass and the choices of any individual firm have no effect on aggregates, and firms employ a measure of workers. Workers can be either employed or unemployed and supply one unit of labor when employed. There is a large number M of local labor markets indexed by m. Where possible, we will disregard the m subscripts and focus on one local labor market at a time since there is no labor mobility across markets.

Search Environment and Contract Space Firms and workers match in a frictional labor market, and matching is random. Workers search with probability one when unemployed and with probability $\lambda_{EE} \in [0,1]$ when employed. Workers receive an amount $b > 0$ from home production when unemployed, and workers can quit into unemployment if they are not matched with an outside job. Firms post wages and are subject to two constraints. First, firms must pay all workers within a cohort the same wage. That is, there is a firm-by-tenure pay equality constraint. As a consequence, firms cannot make individual counter-offers to workers. Second, firms are required to pay at least the posted wage to a given cohort of workers as long as those workers are in the firm.[5] Thus, for a cohort of workers hired by firm j in period s, firms face the constraint $w_{j,s,t} \geq w_{j,s,s}$ $\forall t \geq s$, where $w_{j,s,s}$ is the minimum wage that firm j committed to in period s, and $w_{j,s,t}$ is the wage paid in period t to workers hired in period s. The total number of workers in the firm, $N_{j,t}$, is the sum of all workers from each past cohort still remaining in the firm: $N_{j,t} = \sum_{s=-\infty}^{t} N_{j,s,t}$, whe re $N_{j,s,t}$ is the number of workers hired in period s that still remain in the firm by period t.

Let U_t be the unemployment rate in period t, so the share of employed workers is $1 - U_t$. Firm j's effective recruiting activity in period t is $V_{j,t}$, and aggregate recruiting activity is given by $\bar{V}_t = \int_j V_{j,t} dj$. Workers separate exogenously into unemployment at rate s_0. Labor

[5]This can be rationalized by pay transparency laws that require employers to post wages that are similar to what will be paid in the future.

market tightness is defined as the ratio of vacancies to searchers, $\theta_t \equiv \bar{V}_t/\mathcal{S}_t$, where $\mathcal{S}_t \equiv (1-U_{t-1})(1-s_0)\lambda_{EE} + U_{t-1}$. The share of searchers who are unemployed is $\Phi_{U,t} = U_{t-1}/\mathcal{S}_t$, and the share of searchers who are employed is the complement $\Phi_{E,t} = 1 - \Phi_{U,t}$. Total matches $\mathcal{M}(\bar{V}_t, \mathcal{S}_t,)$ is a constant returns to scale function of aggregate recruiting activity and searchers. Searching workers match with firms with probability $f(\theta_t) \equiv \frac{\mathcal{M}_t}{\mathcal{S}_t}$, and a firm that exerts $V_{j,t}$ units of recruiting activity matches with $V_{j,t}g(\theta_t)$ workers, where $g(\theta_t) = \frac{\mathcal{M}_t}{\bar{V}_t}$. We restrict to functions with $f(\theta_t) \in [0,1]$.

The timing within a period is as follows. The economy inherits the unemployment rate U_{t-1} from the previous period, and each firm j inherits a number of workers from each past cohort $N_{j,s,t-1}$, where each cohort has its own cohort-specific minimum wage $w_{j,s,s}$. At the beginning of the period, exogenous separations occur at rate s_0, and those workers cannot search this period. Then firms choose their wage schedule $\{w_{j,s,t}\}_{s\leq t}$ and recruiting activity $V_{j,t}$, and matching occurs, which determines the employment levels $N_{j,t}$ and unemployment rate U_t for period t. Workers who are not matched may quit into unemployment. At the end of the period, firms produce and pay wages.

2.2 Workers

Workers get indirect flow utility from wage income w and discount at rate β_w. A worker hired in period s by firm j has value $\mathcal{V}_{j,s,t}$ in period t. When choosing between two jobs at firms j and k, workers draw idiosyncratic preference shocks for each job $\iota_{i,j,t}$ from a Type-1 extreme-value distribution with scale parameter γ. The worker then chooses the job that offers the highest utility, solving

$$\max\{\mathcal{V}_{j,s,t+1} + \iota_{i,j,t+1}, \mathcal{V}_{k,t+1,t+1} + \iota_{i,k,t+1}\}.$$

If workers are not exogenously separated and are not choosing between jobs, the worker may voluntarily quit into unemployment. In this case, workers do not receive idiosyncratic preference draws. The worker's value $\mathcal{V}_{j,s,t}$ of working at firm j solves

$$\begin{aligned}\mathcal{V}_{j,s,t} = {} & \frac{\eta}{\eta-1}\left(w_{j,s,t}^{\frac{\eta-1}{\eta}} - 1\right) + \beta_w s_0 \mathcal{V}_U \\ & + \beta_w(1-s_0)\mathbb{E}_t\Big[\lambda_{EE} f(\theta) \int \upsilon_k \, \gamma^{-1} \log\big(\exp(\gamma \mathcal{V}_{j,s,t+1}) + \exp(\gamma \mathcal{V}_{k,t+1,t+1})\big)\, dk \\ & \qquad + \; (1-\lambda_{EE} f(\theta)) \max\{\mathcal{V}_{j,s,t+1}, \mathcal{V}_U\}\Big], \qquad (1)\end{aligned}$$

where $\mathcal{V}_U$ is the value of unemployment, and υ_k is the density of values associated with the posted wage distribution, with distribution $\Upsilon(\mathcal{V})$. Because we solve for a stationary equilibrium, the variables $\mathcal{V}_U$, θ, and υ_k do not have time subscripts, and $\int \upsilon_k \, dk = 1$.

The timing assumptions embedded in equation (1) are as follows: workers produce and receive wage payments at the end of the current period. At the beginning of the next period, workers are exogenously separated with probability s_0. With probability $1 - s_0$, workers retain the option to keep their job. Next, workers are allowed to search on the job with probability λ_{EE}. Conditional on searching, workers match with a hiring firm with probability $f(\theta)$. The density of expected valuations at other firms $\mathcal{V}_k$ is υ_k. Conditional on having matched, workers choose which job offers the highest utility, taking into account the expected future income and the idiosyncratic preference draw. Conditional on having matched with a firm where the expected value is $\mathcal{V}_k$, the probability that the worker who started at firm j in period s accepts the outside offer in period t is

$$\mathbb{P}[\text{worker at firm } j \text{ switches to firm } k] = \frac{\exp\left(\gamma \mathcal{V}_{k,t,t}\right)}{\exp\left(\gamma \mathcal{V}_{j,s,t}\right) + \exp\left(\gamma \mathcal{V}_{k,t,t}\right)}.$$

The higher the value of γ, the more sensitive workers are to differences in values across firms when choosing jobs. Finally, workers who are unemployed have the following value function[6]

$$\mathcal{V}_U = \frac{\eta}{\eta - 1}\left(b^{\frac{\eta-1}{\eta}} - 1\right) + \beta_w \mathbb{E}_t\left[f(\theta)\int_{\mathcal{V}_k \geq \mathcal{V}_U} \upsilon_k \mathcal{V}_{k,t+1,t+1} d\mathcal{V}_k + (1 - f(\theta))\mathcal{V}_U\right].$$

2.3 Firms

Production Firm j in the local labor market m produces a tradable intermediate good $Y_{j,m,t}$. Competitive final goods bundlers produce a final good using a CES production technology: $Y_t = \left(\mathbb{A}_t^{-1} \sum_{m=1}^{M} \sum_{j=1}^{J} A_{j,m,t} Y_{j,m,t}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$ where $\mathbb{A}_t^{-1} = \frac{1}{\sum_{m=1}^{M}\sum_{j=1}^{J} A_{j,m,t}}$.[7] We assume that local labor markets m are sufficiently large that firms do not take into account their effect on the product market.

We will focus on a single local labor market and for now drop the m subscripts. Firms produce with a Cobb–Douglas production technology with $Y_{j,t} = K_{j,t}^{\alpha} N_{j,t}^{1-\alpha}$ using capital $K_{j,t}$ and the measure of workers $N_{j,t}$, with $\alpha \in [0, 1)$. We normalize the price level so that firms face an isoelastic product demand function each period of the form $P_{j,t} = A_{j,t} Y_{j,t}^{-\frac{1}{\epsilon}}$, where the product demand elasticity is $-\epsilon$, and $A_{j,t} = A_j + a_{j,t}$ is a firm-specific product demand shifter that consists of a permanent component A_j and a potentially persistent shock $a_{j,t}$. $a_{j,t}$ evolves according to $a_{j,t} = G(a_{j,t-1}) + z_{j,t}$, where $G()$ is deterministic and known to firms and workers, and $z_{j,t}$ occurs with probability 0 for each firm each period. In total, the firm's

[6] We assume that workers do not take preference draws when matching with jobs from unemployment. Assuming so would not change our core results.

[7] This is equivalent to nested CES where the elasticities of substitution of the inner and outer nests are both ϵ.

revenue is $A_{j,t}\left(K_{j,t}^{\alpha}N_{j,t}^{1-\alpha}\right)^{\frac{\epsilon-1}{\epsilon}}$. Firms can rent capital elastically at rate r^K. Firms may differ in $A_{j,t}$ and c_j, but χ, σ, and α are common parameters.

Worker Flows Firms choose wages period by period, subject to the constraint that the wage is at least the minimum wage posted for each cohort. Firms know that paying higher wages increase worker values, and higher worker values increase the rate of recruiting workers and slow the rate of workers separating. Let $R(\mathcal{V}_{j,t,t})$ be the number of hires per unit of recruiting activity $V_{j,t}$, with $R'(\mathcal{V}_{j,t,t}) > 0$. The value function has two t subscripts because workers who are hired by this firm in period t will have a cohort identifier $s = t$. Also let $S(\mathcal{V}_{j,s,t}) \in [0,1]$ be the share of incumbent workers hired in period s and still employed by that firm at the beginning of period t who separate from the firm, with $S'(\mathcal{V}_{j,s,t}) < 0$. We assume that firms employ a continuum of workers, so there is no uncertainty in a given period about how many workers will be hired or separated conditional on the firm's recruiting activity and worker values. The recruiting rate is

$$R(\mathcal{V}_{j,t,t}) = g(\theta)\left(\Phi_E\left(\int \phi_k \frac{\exp\left(\gamma\mathcal{V}_{j,t,t}\right)}{\exp\left(\gamma\mathcal{V}_{j,t,t}\right)+\exp\left(\gamma\mathcal{V}_{k,s,t}\right)}\,dk\right) + \Phi_U \mathbb{1}\{\mathcal{V}_{j,t,t} \geq \mathcal{V}_U\}\right), \tag{2}$$

where ϕ_k is the density of values that workers are employed at, with $\int \phi_k\, dk = 1$ and distribution $\Phi(\mathcal{V})$. The separation rate is

$$S(\mathcal{V}_{j,s,t}) = s_0 + (1-s_0)\left(\lambda_{EE}f(\theta)\int \nu_k \frac{\exp\left(\gamma\mathcal{V}_{k,t,t}\right)}{\exp\left(\gamma\mathcal{V}_{j,s,t}\right)+\exp\left(\gamma\mathcal{V}_{k,t,t}\right)}\,dk + (1-\lambda_{EE}f(\theta))\mathbb{1}\{\mathcal{V}_U > \mathcal{V}_{j,s,t}\}\right), \tag{3}$$

where ν_k is the density of posted values, with $\int \upsilon_k\, dk = 1$ and distribution $\Upsilon(\mathcal{V})$, as before. Note that $R(\cdot)$ and $S(\cdot)$ do not have time subscripts because there are no aggregate shocks, and we will only solve for a stationary equilibrium.

Recruiting Cost Function Firms begin every period with $N_{j,t-1}$ incumbent workers and can choose a level of effective recruiting activity $V_{j,t}$. Let $\mathcal{C}_{j,t}$ be firm j's expenditure on recruiting in period t. The recruiting function can be expressed in two ways:

$$\mathcal{C}_{j,t} = c_j\left(\frac{V_{j,t}}{N_{j,t-1}}\right)^{\chi} N_{j,t-1}^{\sigma} V_{j,t} \qquad \Leftrightarrow \qquad V_{j,t} = c_{j,0}\,\mathcal{C}_{j,t}^{\frac{1}{1+\chi}}\,N_{j,t-1}^{\frac{\chi-\sigma}{1+\chi}} \tag{4}$$

where σ is unrestricted, $\chi \geq 0$, and $c_{j,0} \equiv c_j^{-1/(1+\chi)}$. χ determines how much marginal recruiting costs increase with the recruiting rate $V_{j,t}/N_{j,t-1}$, while σ governs how firm size affects recruiting costs directly, given the recruiting rate. We will use "effective recruiting activity" and "vacancies" interchangeably, as $V_{j,t}$ is intended to capture all possible activities that firms can engage in to match with more workers. These activities could include instructing current employees to share with their social network that the firm is hiring, recalling former employees, putting help-wanted signs in stores, paying for online job postings, attending career fairs, etc. The assumption behind our recruiting function is that firms rationally choose the least cost per unit of effective recruiting first, and firms choose more expensive forms of recruiting when the cheaper forms of recruiting are exhausted.[8] The second way of formulating equation (4) shows that effective recruiting activity $V_{j,t}$ is a function of two inputs, expenditure and firm size. The recruiting function exhibits constant returns to scale if $\sigma = 0$: doubling both firm size and recruiting expenditure doubles the amount of recruiting activity and doubles the number of matches.[9] If $\chi > 0$, then the firm faces diminishing returns to recruiting expenditure $\mathcal{C}_{j,t}$ for a given firm size $N_{j,t-1}$.

Firm's Problem Firms maximize the present discounted value of revenue less wage, capital, and recruiting costs, with discount factor β_f. Because firms are only subject to idiosyncratic MIT shocks to the demand shifter $A_{j,t}$, we can write the firm's problem in terms of certainty equivalence. Given cohort-level employment $\{N_{j,s,-1}\}_{s\leq -1}$, and the history of posted wages $\{w_{j,s,s}\}_{s\leq -1}$, firms solve

$$\max_{\left\{V_{j,t},\, K_{j,t},\, N_{j,t},\, \{N_{j,s,t}\}_{s\leq t},\, \{w_{j,s,t}\}_{s\leq t}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_f^t \left(A_{j,t} \left(K_{j,t}^{\alpha} N_{j,t}^{1-\alpha}\right)^{\frac{\epsilon-1}{\epsilon}} - \sum_{s=-\infty}^{t} w_{j,s,t} N_{j,s,t} - r^K K_{j,t} - c_j \left(\frac{V_{j,t}}{N_{j,t-1}}\right)^{\chi} N_{j,t-1}^{\sigma} V_{j,t} \right) \quad (5)$$

subject to

$$\text{Recruiting:} \qquad N_{j,t,t} = R(\mathcal{V}_{j,t,t})\, V_{j,t} \quad \forall\, t \quad (6)$$

$$\text{Retention:} \qquad N_{j,s,t} = (1 - S(\mathcal{V}_{j,s,t}))\, N_{j,s,t-1} \quad \forall\, t \text{ and } s < t \quad (7)$$

$$\text{Employment is the sum of cohorts:} \qquad N_{j,t} = \sum_{s=-\infty}^{t} N_{j,s,t} \quad \forall\, t \quad (8)$$

$$\text{Wage at least posted wage:} \qquad w_{j,s,t} \geq w_{j,s,s} \quad \forall\, t \text{ and } s \leq t, \quad (9)$$

[8]Prior research has documented the importance of accounting for recruiting intensity, beginning with Davis et al. (2013) and more recently by Gavazza et al. (2018) and Mongey and Violante (2025). These papers distinguish between vacancies and recruiting intensity. We sidestep this distinction, as our measure of "vacancies" is a reduced-form composite of all possible recruiting activities.

[9]Modeling matches as increasing in firm size is supported by evidence that many hires are done through referrals and social networks: see Burks et al. (2015), Caldwell and Harmon (2019), Jahn and Neugart (2020) and Dustmann et al. (2016).

where (1), (2), and (3) are the worker's value function, the recruiting rate, and the separation rate, respectively. For simplicity, we will focus on the limiting case where firms do not discount ($\beta_f \to 1$) for the remainder of the paper.[10]

2.4 Stationary Equilibrium

A stationary equilibrium consists of a mass of searchers $\mathcal{S}$, labor-market tightness θ, an unemployment rate U, distributions of values across employed workers $\Phi(\mathcal{V})$ and across vacancies $\Upsilon(\mathcal{V})$, an employment policy $\{N^*_{j,s,t}\}_{s\leq t}$, a wage policy $\{w^*_{j,s,t}\}_{s\leq t}$, and a recruiting policy $\{V^*_{j,t}\}$ such that (i) firms maximize profits, (ii) workers maximize utility, and (iii) worker inflows and outflows balance each period. We focus on equilibria with two desirable properties. First, in steady state, firms choose a constant wage profile for each cohort: $w_{j,s,t} = w_{j,s,s}$ for all $t \geq s$.[11] Second, firms of a given type, characterized by their firm-specific parameters (A_j, c_j), have a unique optimal steady-state wage.

Constant wages arise as an equilibrium result from two factors: (i) firms have more monopsony power over incumbents than over matched workers who have not accepted a job yet, and (ii) firms are constrained to pay at least the posted wage. Firms have greater monopsony power over incumbent workers because many incumbent workers are not searching on the job in any given period and have only unemployment as an outside option. In contrast, many potential new hires are searching on the job and have their current job as an outside option. Once workers accept a job and become less mobile, firms would find it profitable to renege and cut wages. Since the wage-posting constraint prevents this, the best remaining option for firms is to pay constant wages.[12] We verify numerically that firms have no incentive to deviate from a constant wage path in steady state under a wide range of plausible parameter values,[13] and we prove it formally for a special case in Section 2.7.

Now that we restrict to a stationary equilibrium with constant wage–tenure profiles (so

[10]We show in Appendix B.3 that when this model is calibrated to standard monthly parameters, the discount factor is quantitatively unimportant for profits.

[11]Specifically, it is optimal for firms to choose a fixed wage profile under the belief that workers expect firms will revert to the posted wage if firms deviate. See Coles (2001) and Manning (2025) for wage setting without commitment and Stevens (2004) for contracting with commitment.

[12]The alternative strategy is to back-load wages: post a low starting wage and raise wages later for incumbents. However, since firms cannot credibly commit to future wages above the posted minimum, workers anticipate reversion to the posted wage. Back-loading is therefore a costly way to increase retention and is not desirable for the firm.

[13]That is, we verify that parameterizations that even loosely match the empirical unemployment rate, job-to-job transition rate, vacancy rate, and recruiting and separation elasticities generate flat wage-tenure profiles in equilibrium.

$w_{j,s,t} = w_{j,s,s}$ for all $t \geq s$), we can define a firm's recruiting and separation elasticities, respectively denoted ε_{R,w_j} and ε_{S,w_j}. The recruiting elasticity is $\varepsilon_{R,w_j} \equiv \frac{R'(w_j)w_j}{R(w_j)}$, where the derivative of the recruiting function $R'(w_j) = \frac{\partial R}{\partial \mathcal{V}_j}\frac{\partial \mathcal{V}_j}{\partial w_j}$ is taken under the mutual expectation by both workers and firms that the wage will be constant throughout the duration of the employment relationship. Analogously, the separation elasticity is defined as $\varepsilon_{S,w_j} \equiv \frac{S'(w_j)w_j}{S(w_j)}$ with $S'(w_j) = \frac{\partial S}{\partial \mathcal{V}_j}\frac{\partial \mathcal{V}_j}{\partial w_j}$, where $S'(w_j)$ captures how a cohort's separation rate responds to the posted wage. When posting a wage, the firm takes into account that paying a higher wage will accelerate the filling of a vacancy and reduce the rate of separations for that cohort of workers. Faster hiring and slower separations reduce the number of vacancies V_j the firm needs to post, decreasing its recruiting costs. The firm's optimal steady-state wage satisfies

$$w_j = c_j(1+\chi)\left(\frac{S(w_j)}{R(w_j)}\right)^{1+\chi}\left(\varepsilon_{R,w_j} - \varepsilon_{S,w_j}\right)N_j^{\sigma}. \tag{10}$$

Since $S'(w) < 0$ and $S(w) > 0$, the separation elasticity is negative, $\varepsilon_{S,w} < 0$. The steady-state ratio of vacancies to employment takes the form

$$\frac{V_j}{N_j} = \frac{S(w_j)}{R(w_j)}. \tag{11}$$

Together, equations (10) and (11) demonstrate that the firm trades off wage costs and turnover costs: a higher wage means that the firm will post fewer vacancies in steady state, decreasing recruiting costs. Conversely, a low wage implies higher steady-state turnover costs. We can define the sum of recruiting and (negative) separation elasticities as

$$\mathcal{E}(w_j) \equiv \varepsilon_{R,w_j} - \varepsilon_{S,w_j}.$$

For presentation purposes, we will suppress the argument (w_j), and simply write this sum of elasticities as $\mathcal{E}_j$. The optimal steady-state level of employment for firm j is

$$N_j = \left(\frac{\frac{\epsilon-1}{\epsilon}(1-\alpha)A_j}{w_j}\right)^{\epsilon}\left(\frac{w_j}{r^K}\frac{\alpha}{1-\alpha}\right)^{\alpha(\epsilon-1)}\left(\frac{\mathcal{E}_j(1+\chi)}{1+\mathcal{E}_j(1+\chi)+\sigma}\right)^{\epsilon-\alpha(\epsilon-1)}. \tag{12}$$

Finally, the second convenient property that emerges from this model is that, under a broad range of reasonable parameterizations, ex-ante identical firms have a unique and identical profit maximizing wage. If all firms are ex-ante identical, then the equilibrium features a point mass of wages. This stands in contrast to the famous result in Burdett and Mortensen (1998), that, with on-the-job search and ex-ante homogeneous workers and firms, the equilibrium must contain wage dispersion. As in Albrecht et al. (2018), we achieve this by workers having horizontally differentiated preferences over workplaces. This means that even if all firms are ex-ante identical and pay a common wage, workers will have non-degenerate outside value distributions. This means that the probability that a worker leaves a given job is a smooth function of their current wage and a competing outside wage, creating a smooth trade-off for firms between wage costs and turnover probabilities. If identical firms are facing the same smooth trade-off, then all firms will choose the same wage, generating the point mass of wages in equilibrium.[14] [15]

Characterizing $\mathcal{E}_j \equiv \varepsilon_{R,w_j} - \varepsilon_{S,w_j}$ The equilibrium values of each firm's recruiting and separation elasticities depend on many equilibrium objects, including the firm's wage, the distribution of earned wages, the distribution of posted wages, the typical duration of employment, and the unemployment rate, all of which are complex functions of parameters. The parameter that most influences the recruiting and separation elasticities is γ, the parameter that governs the variance of the Type-1 extreme value preference draws. When γ is higher, the variance of these draws is lower, making relative wages more important in workers' decisions to move across firms, which makes workers more wage sensitive and raises $\varepsilon_{R,w}$ and $-\varepsilon_{S,w}$ all else equal, and vice-versa. Empirical evidence (Bassier et al., 2022; Datta, 2023; Sokolova and Sorensen, 2021) tends to find recruiting and separations elasticities to be around 2 and -2, respectively, with the sum typically around 4. When calibrating the model, we treat γ as a free parameter to make $\varepsilon_{R,w} - \varepsilon_{S,w}$ close to 4, and this will be exact in a special case in Section 2.7.

2.5 Decomposing the Marginal Product

At the firm level, we decompose the marginal product of labor into the shares that go towards wages, recruiting costs, and profits. The expressions of these shares are reported in Table 1. Each expression depends on the sum of recruiting and separation elasticities $\mathcal{E}_j$ and the recruiting cost parameters χ and σ. The first three rows of the table simply represent a decomposition: what share of marginal product is spent on wages, spent on per worker recruiting, and what remains. We label the share of marginal product that is left

[14] We verify numerically that no frictional wage dispersion is an equilibrium result for reasonable parameters values. The no-dispersion equilibrium can break down when the variance of preference draws becomes sufficiently small (i.e., γ becomes sufficiently large). However, this typically requires a sum of recruiting and separation elasticities over 10, far above the empirically estimated range. We prove unique optimal wages for the special case outlined in Section 2.7 in Appendix B.5.4.

[15] In addition to helping with tractability, idiosyncratic preference draws allow for workers to voluntarily take wage cuts when moving between jobs (Sorkin, 2018; Hall and Mueller, 2018) in any equilibrium with wage dispersion.

after accounting for wage and recruiting costs "labor market profits."[16]

Table 1: Decomposing Marginal Product into Wages, Recruiting Costs, and Profits

Outcome	Definition	Formula
Wage share of *MRPL*	$\frac{w_j}{MRPL_j}$	$\frac{(1+\chi)\mathcal{E}_j}{1+(1+\chi)\mathcal{E}_j+\sigma}$
Recruiting cost share of *MRPL*	$\frac{\text{Recruiting costs per worker}_j}{MRPL_j}$	$\frac{1}{1+(1+\chi)\mathcal{E}_j+\sigma}$
Labor market profit share of *MRPL*	$\frac{MRPL_j - w_j - \text{recruiting cost per worker}_j}{MRPL_j}$	$\frac{\sigma}{1+(1+\chi)\mathcal{E}_j+\sigma}$
Inverse labor supply elasticity	$\varepsilon_{w_j,N_j} \equiv \frac{\partial \log w_j/\partial \log A_j}{\partial \log N_j/\partial \log A_j}$	$\frac{\sigma}{1+(1+\chi)\mathcal{E}_j-\varepsilon_{\mathcal{E}_j,w_j}}$
Recruiting cost-adjusted wage markdown	$\frac{w_j + \text{recruiting cost per worker}_j}{MRPL_j}$	$\frac{1+(1+\chi)\mathcal{E}_j}{1+(1+\chi)\mathcal{E}_j+\sigma}$

This table shows the wages, recruiting cost, and labor market profit shares of marginal product, the recruiting cost-adjusted markdown, and the inverse labor supply elasticity. Analogous results for the general β_f case can be found in Appendix B.3. Recruiting cost per worker is defined as $\mathcal{C}_j/N_j \equiv c_j\left(V_j/N_j\right)^{1+\chi}N_j^{\sigma} = \mathcal{C}_j/H_j \times S_j$, which is equivalent to the costs per hire times the separation rate, where H_j is the number of hires with $H_j \equiv R_j \times V_j$.

The fourth term in Table 1 is the inverse labor supply elasticity, which shows the elasticity of the optimal wage with respect to the firm's optimal employment level changes in response to a permanent change in firm specific product demand A_j. Within the inverse labor supply term is a super-elasticity $\varepsilon_{\mathcal{E}_j,w_j}$, which is the elasticity of $\mathcal{E}_j$ with respect to w_j. This super-elasticity captures that the sum of recruiting and separation elasticities may change with the wage. The final term in the table is the recruiting cost-adjusted wage markdown, which is the share of marginal product that is spent by the firm on wages and recruiting.[17]

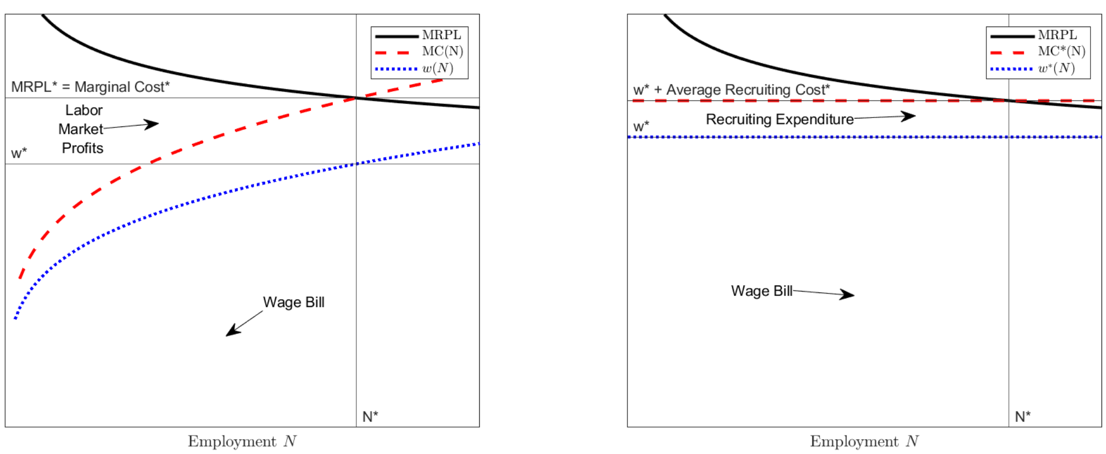
[16]Note that the marginal benefit of hiring a worker includes the present discounted value of future marginal products, as well as lower costs of future recruiting. Firms equalize the marginal benefit and marginal cost of a hire, which can be expressed as the following when firms do not discount: $\frac{w}{S}+(1+\chi)\frac{\mathcal{C}/V}{R} = \frac{MRPL}{S} - \frac{(\sigma-\chi)\mathcal{C}/N}{S}$.

[17]In our base case, we assume that recruiting costs are an expenditure that accrues to labor income. An alternative is that recruiting costs are subtracted from output, in which case the recruiting cost-adjusted

Two parameterizations are of particular interest. The first is the limiting case in which the recruiting margin is shut down as a choice variable. This occurs when $\sigma = \chi$ and $\sigma, \chi \to \infty$. Plugging these parameter values into Table 1, we obtain the standard dynamic monopsony model without a recruiting margin, where the labor supply elasticity $\varepsilon_{N_j,w_j} = \mathcal{E}_j \equiv \varepsilon_{R,w_j} - \varepsilon_{S,w_j}$, the firm spends nothing on recruiting, and the wage markdown is $\frac{\mathcal{E}_j}{1+\mathcal{E}_j}$. Thus, our model with a recruiting margin nests standard models of dynamic monopsony without a recruiting margin as a special case, including static models of monopsony without a recruiting margin. Figure 1a depicts this standard case graphically: the firm's marginal cost curve lies above the labor supply curve, and the marginal product intersects marginal cost at a point where wages are below marginal product.

Figure 1: Wage Bill, Recruiting Costs, and Marginal Product

(a) Standard Monopsony: $\sigma = \chi$, $\sigma, \chi \to \infty$ (b) Constant Returns to Scale Recruiting: $\sigma = 0$



This figure shows the marginal product, marginal cost, and labor supply curves for firms with different recruiting functions. Panel (a) shows a firm in a standard monopsony model with no recruiting function, equivalent to $\sigma = \chi$ and $\sigma, \chi \to \infty$ in our model. The firm's marginal cost line lies above the firm's labor supply curve, and the wage lies below marginal product when the marginal product curve and marginal cost curve intersect. Panel (b) shows a firm with a constant returns to scale recruiting function. For this firm, the optimal wage is the same regardless of firm size, and the share of recruiting costs in total costs is also constant across different choices of steady state firm size. The wage lies below marginal product at the optimum, with the gap entirely absorbed by recruiting costs.

The second parameterization of interest is $\sigma = 0$, in which hiring costs depend only

wage markdown would be $w/(MRPL -$ recruiting costs per worker). A third alternative is that recruiting effort may divert labor away from productive activity, such that labor inputs are $N - cV^{1+\chi}N^{-(\chi-\sigma)}$, which we explore in Appendix B.4. These different assumptions do not affect our core results.

on the recruiting rate (vacancies per worker, V/N) and not directly on firm size. This parameterization implies an inverse labor-supply elasticity of zero and that the recruiting-cost–adjusted wage markdown equals 1: even though wages lie below marginal product, the gap between wages and the marginal product is entirely absorbed by recruiting costs. This is depicted in Figure 1b. Within this $\sigma = 0$ case, there are two subcases: $\chi > 0$ and $\chi = 0$. In the former case, marginal hiring costs increase with the recruiting rate (V/N), which encourages higher wages and discourages vacancy posting, pushing up the wage share of marginal product and pushing down the average recruiting cost per worker. Under $\chi = 0$, recruiting costs are linear in the number of vacancies.[18]

2.6 Evidence on Recruiting Costs

Our emphasis on recruiting costs raises a natural question: are recruiting costs large enough to account for the difference between wages and marginal products, and is there evidence of diminishing returns to recruiting expenditure?

To begin, the ratio of wages to marginal product implied by a model with recruiting can be much closer to 1 than would be implied by a model without a recruiting margin. As a consequence, the level of recruiting costs does not need to be large in order for the recruiting cost-adjusted markdown to be equal to 1. To see this, consider the evidence that the sum of recruiting and separation elasticities $\mathcal{E} \equiv \varepsilon_{R,w} - \varepsilon_{S,w}$ is typically estimated to be around 4 (Bassier et al., 2022; Datta, 2023; Sokolova and Sorensen, 2021). In a model without a recruiting margin ($\sigma = \chi$, $\sigma, \chi \to \infty$), this would imply a wage share of marginal product equal to $\frac{\mathcal{E}}{1+\mathcal{E}} = 0.8$. Consider instead a different benchmark of a constant returns to scale recruiting function ($\sigma = 0$) but firms face convexity in recruiting costs as a function of vacancies per incumbent, with $\chi = 2$ as proximately the value that we estimate for χ. In that case, holding fixed the recruiting minus separation elasticities $\mathcal{E} \equiv \varepsilon_{R,w} - \varepsilon_{S,w} = 4$, the ratio of the wage to marginal product becomes $\frac{\mathcal{E}(1+\chi)}{1+\mathcal{E}(1+\chi)+0} \approx 0.92$. Thus, a higher value of χ pushes wages closer to marginal product, holding $\varepsilon_{R,w} - \varepsilon_{S,w}$ fixed.

When interpreting evidence on turnover or hiring costs, it is important to distinguish between average and marginal hiring costs. In our setting, the marginal hiring cost equals $(1+\chi)$ times the average hiring cost. Evidence on average hiring costs suggests they are moderate: Manning (2011) surveys the literature and Dube et al. (2010) document average hiring costs

[18] In Appendix C.4 we extend the model so firms can invest in costly amenities but shut down the recruiting margin, showing that in such a model, an infinitely elastic long run labor supply curve is a knife-edge case that requires there be no exogenous separations. Thus an amenities margin is not isomorphic to a recruiting margin, as offering better amenities only raise the probability that a worker accepts a job but does not help firms overcome search frictions.

of around 2-5% of the wage bill. Blatter et al. (2012) find the average cost of replacing skilled workers to be 10–17 weeks of salary. Michaillat and Saez (2024) argue that servicing a vacancy requires approximately one full-time worker. In terms of marginal hiring costs, Muehlemann and Pfeifer (2016) find that marginal hiring costs are increasing, consistent with $\chi = 1.3$. Bloesch and Weber (2023) find evidence of congestion in onboarding software developers, consistent with convex costs in the rate of training new workers. Evidence on separation costs, and by implication the marginal cost of a hire, tends to be larger: Kline et al. (2019), Jäger and Heining (2022), Bertheau et al. (2021), and Bloesch et al. (2022) all point to separation costs of 1–3 years of a workers' wages. However, the latter three papers estimate these costs from worker deaths, which could be significantly more costly than replacing a worker after a typical quit.[19] Models that calibrate an object like χ in order to match empirical relationships between vacancy rates and hiring rates imply values of χ between 1 and 8 (Davis et al., 2013; Kaas and Kircher, 2015; Gavazza et al., 2018; Mongey and Violante, 2025).

Our model has strict implications for the present value of a match to a firm and the level of marginal hiring costs in steady state. Given the expressions for wages and recruiting costs in Table 1, the recruiting expenditure per incumbent each period in steady state is $\frac{w}{\mathcal{E}(1+\chi)}$. In steady state, the number of hires equals the number of separations SN, so the recruiting expenditure per hire is $\frac{w}{S\mathcal{E}(1+\chi)}$. Given the convexity parameter χ, the marginal hiring cost equals $(1 + \chi)$ times the average cost per hire, so the marginal hiring cost is $\frac{w}{S\mathcal{E}}$. Therefore, with $\mathcal{E} = 4$ and a monthly separation rate between 0.04 and 0.06,[20] this would imply a marginal hiring cost per hire of around 4–6 months of wages. To be consistent with evidence on the level of hiring costs, this would imply a value of χ roughly between 1 and 3, which is consistent with our estimates in the next section.

2.7 Special Case: Isoelastic Labor Supply Curves

We now highlight a highly tractable special case that closely relates our model to existing models of monopsony.

[19] We consider hiring costs that do not vary with wages or labor market tightness, such as training costs, in Appendix C.3. For these costs, higher wages save turnover costs only by preventing separations, resulting in more weight on the separation elasticity and less weight on the recruiting elasticity when setting wages.

[20] The rate of worker flows across firms is similar Denmark and in the US. Caldwell and Harmon (2019). Bagger et al. (2022) find that the monthly separation rate in Denmark is around 6.6 percent. The total monthly job separation rate from the Job Opening and Labor Turnover Survey (JOLTS) in the US averages between 3 and 4 percent.

Proposition 1. *If workers have log utility over consumption ($\eta \to 1$), there are no exogenous separations ($s_0 = 0$), workers fully discount the future ($\beta_w \to 0$), and σ is not too negative, then there exists a unique stationary equilibrium such that:*

1. *firms' optimal wage is constant in steady state,*
2. *ex-ante identical firms pay a common wage, and*
3. *the recruiting elasticity minus the separation elasticity $\mathcal{E}_j \equiv \varepsilon_{R,w_j} - \varepsilon_{S,w_j}$ is constant and is equal to γ for any w_j.*

Proof: See Appendix B.5. This case is particularly useful because when we shut down the recruiting margin ($\sigma = \chi$ and $\sigma, \chi \to \infty$), the model in steady state is isomorphic to static models where firms face an isoelastic residual labor supply curve, as is the case in Lamadon et al. (2022) without amenities, in Card et al. (2018) when $b = 0$ (in their notation), and in Berger et al. (2022) when firms are atomistic. By nesting this common special case, we can more clearly isolate how introducing a recruiting margin generates different implications relative to this important benchmark for markdowns and profits from wage setting power.

2.8 Wage and Employment Responses to Product Demand Shocks

In this section, we derive the response of wages to idiosyncratic firm demand shocks, which we use in Section 3 to estimate the recruiting cost parameters χ and σ, and by extension estimate the inverse labor supply elasticity $\varepsilon_{w,N}$ and the share of marginal product retained as profits.

We consider how a firm's optimal wage responds to an idiosyncratic shock to $z_{j,t}$, i.e., an idiosyncratic product demand shock. Define $h_{j,t}$ as the firm's hiring rate (hires per incumbent): $h_{j,t} \equiv V_{j,t} R(\mathcal{V}_{j,t,t})/N_{j,t-1}$. Log-linearizing the firm's problem around a steady state with no wage dispersion from Section 2.7, the firm's optimal deviation of new hire wages from steady state is

$$\tilde{w}_{j,t} = \frac{1}{2\Omega}\left(\underbrace{(\chi \tilde{h}_{j,t} + \sigma \tilde{N}_{j,t-1})}_{\substack{\text{Current hiring rate and} \\ \text{prior employment level}}} + S \sum_{\kappa=0}^{\infty} (1-S)^{\kappa} \underbrace{\chi \tilde{h}_{j,t+\kappa+1} + \sigma \tilde{N}_{j,t+\kappa} - (1+\chi)\frac{\gamma}{2}\tilde{w}_{j,t+\kappa+1}}_{\substack{\text{Future hiring rates, employment levels,} \\ \text{and posted wages}}} \right), \quad (13)$$

where tildes denote log deviations from the steady state, $\Omega = \left(1 + \frac{\gamma}{2} + \frac{\gamma(1+\chi)}{4}\right)$, and $S \in (0,1)$ denotes the steady-state separation rate.

The coefficient on hiring rate is increasing in χ: when firms face diminishing returns to recruiting expenditure, matching with an additional worker becomes more costly. Given a limited pool of matched workers, firms raise wages to accelerate hiring, and this incentive is stronger when the marginal match is more expensive. Thus, a higher χ amplifies the firm's optimal wage response for a given hiring target. The coefficient on firm size is increasing in σ: a higher value of σ means that hiring costs increase with firm size, when holding the vacancy rate fixed; thus, leading to a higher optimal wage with larger firm size.

The intuition for identification is that in response to an idiosyncratic demand shock, the firm will respond with a path of wages, hiring rates, and employment levels. Identification of χ and σ will come from whether the wages of new hires co-vary with the hiring rate or the firm's employment level. That is, if wages are related to (recent and future) firm size, then that is evidence that $\sigma > 0$. If instead the wage of new hires co-moves with the hiring rate but not firm size, then that is evidence that $\chi > 0$ but $\sigma = 0$. Lastly, if wages of new hires do not change at all as the firm hiring rate and employment level move around, then that is evidence that $\chi = 0$ and $\sigma = 0$. Differing variation in hiring rates and employment arises because hiring is a flow, which responds quickly to a shock, while firm size is a stock, which lags. In the next section, we will estimate the empirical impulse responses of switcher wage growth, hiring rates, and employment levels at exporting firms to idiosyncratic firm export demand shocks in Denmark, and we will use these results to estimate χ and σ.[21]

3 Empirical Evidence and Parameter Estimation

In this section, we conduct two empirical exercises. First, we study the effect of persistent firm-level export demand shocks on Danish manufacturing firms' employment, hiring rates, and the wage growth of job switchers and stayers. This exercise allows us to estimate the key parameters of our model: the recruiting cost function parameters, χ and σ, and the inverse labor supply elasticity, $\varepsilon_{w,N}$. Second, we document that the wage premium at large firms is small across industries in Denmark. This result provides suggestive evidence that firms can become large without paying high wages in sectors outside of manufacturing.

[21] Our estimation of χ and σ is not dependent on log-linearizing around a steady state without unemployment and with myopic workers. In Appendix C.1, we calibrate the model with forward-looking workers and allow for unemployment. We argue that the estimates of σ and χ from the main text are consistent with a model calibrated with forward-looking workers.

3.1 Data

We construct an annual employer-employee matched dataset using Danish administrative data for the period 2001-2019. The administrative datasets we use are maintained by Statistics Denmark. For workers' employment history and wages, we use the registers in the Integrated Database for Labor Market Research (IDA), which reports workers' earnings, hours worked, occupation, employer, and demographic information. We add information about the workers' education from the education register (UDDA). We link the labor market data with firm characteristics from the Danish business register (FIRM), including revenue, employment, and NACE industry classification. We measure employment in full-time equivalents (FTEs) throughout, and we use two-digit industry codes in our baseline analysis. We use customs and trade data from two sources to construct our firm-level export demand shocks, in two steps. First, we obtain data on the goods exports of Danish firms by product and destination country from the customs data register (UHDM). We use 4-digit Harmonized System (HS) product codes. Second, we use data on international trade flows from the United Nations Comtrade database to measure each country's imports by product and partner source country.

Sample Our main estimation sample is a panel of exporting manufacturing firms. We drop observations with less than 5 employees or revenue below DKK 1 million. In our baseline regression specification, we control for lags of the export demand shock. Including lagged values imposes a balance requirement on our sample: firms must be observed in the data for at least five consecutive years: the year of the shock and the four preceding years. To maintain a consistent sample throughout the paper, we restrict further slightly by including only firms that have non-missing export demand shocks each of the 4 years prior to the shock. Our baseline sample thus includes shocks in years 2006–2019.

The estimation sample consists of 22,453 firm-year observations with 2,961 unique firms. We report descriptive statistics in Table 2. Denmark is a small and very open economy, so most manufacturing firms are exporters, and our sample includes many small- and medium-sized firms. The median firm size is 38 employees, but the distribution is heavily right-skewed with mean employment at 114 employees and several firms of considerable size at the very top. The firms in our sample produce multiple products and export to multiple countries. On average, firms export 13 different products (at the level 4-digit HS codes) and export to 8 destination countries.

Table 2: Firm Summary Statistics

	Mean	Median	10th Percentile	90th Percentile
Employment (FTE)	114.1	37.6	10.0	200.6
Revenue (DKK Mn)	333.8	61.0	13.9	465.0
Exports (DKK Mn)	130.6	20.3	0.9	211.5
Export destinations	17.4	13.0	2.0	39.4
Exported products (4-digit HS)	13.0	8.0	2.0	28.5
Hiring Rate	18.9	15.5	2.9	37.8
Switcher Wage Growth	3.9	3.7	−15.6	23.2
Observations	22,453	No. of firms		2,961

Notes: Sample is exporting manufacturing firms observed in 2006–2019. Employment is measured in full-time equivalents (FTE). Revenue and exports are in DKK millions. "Export destinations" counts distinct destination countries; "Exported products" counts distinct 4-digit HS product lines. "Hiring rate" is $H_{j,t}/N_{j,t-1}$, where $H_{j,t}$ are new hires in year t (workers on the firm's November payroll in t, not employed by the firm in $t-1$ or $t-2$, and with ≥ 50 hours in t). "Switcher wage growth" is the firm-year mean two-year log wage change for new hires who worked at another firm two years earlier: $\Delta \log w_{j,t}^{\text{switcher}} = \frac{1}{|\text{switchers}_{j,t}|} \sum_{i \in \text{switchers}_{j,t}} \left(\log w_{i,j,t} - \log w_{i,k,t-2} \right)$. Percentiles are across firm-year observations; "Observations" counts firm-years and "No. of firms" counts unique firms in the estimation sample.

Table 3: Switching Worker Summary Statistics

	Mean	Median	10th Percentile	90th Percentile
2 Year Log Wage Growth	5.9	4.7	−21.4	33.3
Wage (kr/hr)	208.9	190.6	134.1	299.6
Educ Years	14.4	14.0	11.0	18.0
Age	41.8	42.0	29.0	54.6
Share Male	0.7			
Observations	131973	No. of workers		115228

Notes: Sample includes job switchers (workers moving from employer k to j) in 2006-2019 who are full-time ($\geq 1,400$ annual hours) at both the prior and new employer. "2-year wage growth" is $\log w_{i,j,t} - \log w_{i,k,t-2}$. "Wage (kr/hr)" is hourly earnings in Danish kroner. "Educ Years" comes from the education registry; "Age" is in years; "Share Male" is the fraction male. Percentiles are across worker-level switch events; "Observations" counts worker-year switches and "No. of workers" counts unique individuals.

Table 3 reports summary for statistics for the sample of workers who switch into our sample of export manufacturing firms. The sample is 70% male, and the typical worker has 14 years of education and is 42 years old. The average hourly wage is 190.6 DKK/ hour (approximately $30/hour), and the average worker sees an hourly wage change of 4.7 log points when changing jobs. There are 115,228 unique individuals switching jobs in our sample and 131,973 switcher-year events.

3.2 Construction of Export Demand Shocks

Our goal is to estimate the recruitment cost parameters χ and σ, which in turn will be informative about the inverse labor supply elasticity $\varepsilon_{w,N}$. For this purpose, we need a firm-specific product demand shock that is persistent and plausibly exogenous.

We construct shift-share export demand shocks to Danish exporting firms drawing on Hummels et al. (2014). The idea is that Danish exporters are differentially exposed to changes in foreign demand depending on the products they export and the countries they export to. The shift-share shock variable consists of exposure shares and foreign demand shifters. We construct the demand shifters as follows. For each country-product pair, we compute the world import demand $WID_{c,p,t}$, which is country c's imports of product p from all countries excluding Denmark. We then compute the one-year change in log world import demand $d\log WID_{c,p,t} \equiv \log WID_{c,p,t} - \log WID_{c,p,t-1}$, for each country–product–year cell.

A Danish firm j is exposed to a shift in foreign demand $d\log WID_{c,p,t}$ if they export product p to country c. We define the exposure shares as

$$\omega_{j,c,p,t-1} = \frac{exports_{j,c,p,t-1}}{revenue_{j,t-1}},$$

which captures how exposed firm j's total revenue is to changes in foreign demand for a given country-product pair (c, p). We measure the shares in the previous year. Finally, we combine the shares and the shifters to construct our firm-level shift-share export demand shock:

$$z_{j,t} = \sum_{c,p} \omega_{j,c,p,t-1} d\log WID_{c,p,t}.$$

If firms have domestic sales, then the shares do not add up to one when we sum across countries and products. Instead, they add up to the (lagged) export share of revenue,

$$\Omega_{j,t-1} = \sum_{c,p} \omega_{j,c,p,t-1}.$$

If the export share of revenue varies across firms in a way that is correlated with unobservables, then our estimates will be biased. To deal with this problem, we follow Borusyak et al. (2022) and control for the sum of the export shares. Specifically, since our main specification includes industry-by-year fixed effects, we control for the interaction between the industry-year fixed effects and the lagged export share $\Omega_{j,t-1}$.

Idiosyncratic Export Demand Shocks A potential concern is that the export demand shocks may not be completely idiosyncratic due to global product shocks. Imagine there is a global increase in demand for a specific product p'. All Danish firms that export this product will be affected, and if these firms compete for workers in the same local labor market, then wages may rise more than if only one firm was hit by a truly idiosyncratic shock. If our industry classification is a good approximation of the local labor markets that firms hire in, then industry-by-year fixed effects can absorb the common component of the shocks. However, it is difficult to know whether this is the case in practice. To isolate the idiosyncratic component of the export demand shock, we carry out a decomposition of the demand shifters proposed by Garin and Silvério (2024). We can write the shifters as

$$d\log WID_{c,p,t} = \underbrace{\overline{d\log WID}_{p,t}}_{\text{common component}} + \underbrace{\left(d\log WID_{c,p,t} - \overline{d\log WID}_{p,t}\right)}_{\text{idiosyncratic component}},$$

where $\overline{d\log WID}_{p,t} \equiv \frac{1}{|C|}\sum_{c\in C} d\log WID_{c,p,t}$ is the mean change in log world import demand across countries (with C denoting the set of destination countries, excluding Denmark). In other words, we decompose the foreign demand shifter in year t into a product fixed effect and a residual. The product fixed effect is the "common component", which reflects global changes in demand for p. The residual term is the "idiosyncratic component" of the shifter, which captures changes in the demand for a product p from a destination country c *relative* to the global average change in demand for p. The decomposition is

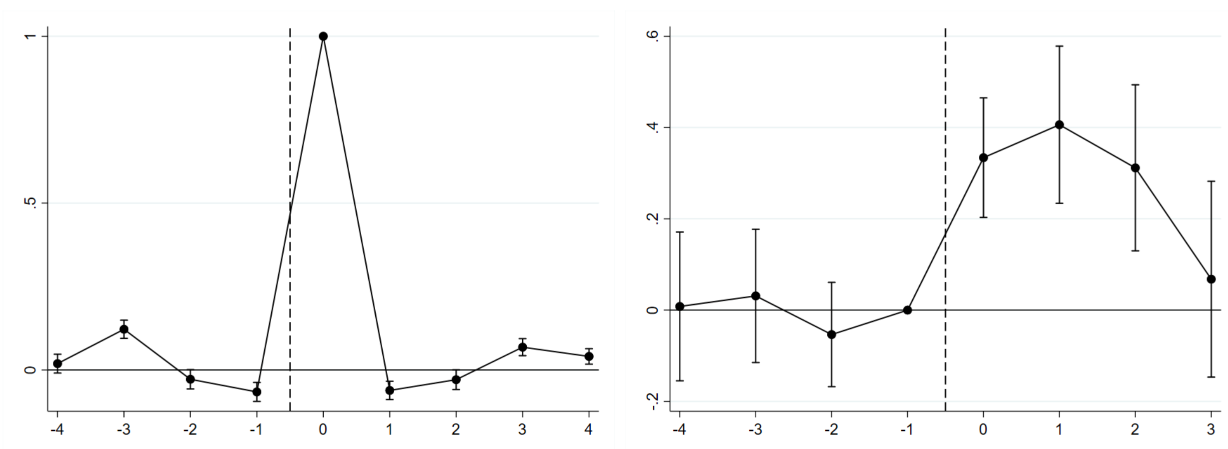
$$z_{j,t} = \underbrace{\sum_p \bar{\omega}_{j,p,t-1}\overline{d\log WID}_{p,t}}_{\equiv z_{j,t}^{common}} + \underbrace{\sum_{c,p} \omega_{j,c,p,t-1}\left(d\log WID_{c,p,t} - \overline{d\log WID}_{p,t}\right)}_{\equiv z_{j,t}^{idiosyncratic}},$$

where $\bar{\omega}_{j,p,t-1} \equiv \sum_{c\in C} \omega_{j,c,p,t-1}$ are product exposure shares. We then winsorize the idiosyncratic export demand shocks at the 1st and 99th percentiles in each year.

Shock Diagnostics Figure 2 shows the diagnostics of our idiosyncratic export demand shock. Figure 2a reports the results of regressing the leads and lags of the shock on the time t shock, including industry-by-year fixed effects. These shocks exhibit minimal autocorrelation (Dhyne et al., 2022). Figure 2b estimates the effect of an idiosyncratic export demand shock on firm log revenue using local projections (see equation Section 3.3 and equation (14) for a discussion of the specification). The export demand shocks predict increases in actual revenue: a 100 log point predicted increase in revenue on average results in a 40 log point increase in actual revenue as shown in Figure 2b. On average, export demand shocks are relatively small: the standard deviation of shocks is 0.046. This means that in a typical year, an exporting firm experiencing a one-standard deviation shock in export demand should expect actual revenue to increase by around 2 percent.

Figure 2: Export Demand Shock Autocorrelation and Revenue Response

(a) Trade Shock Autocorrelation (b) Response of Log Revenue to Trade Shock



Panel (a) shows the coefficients from a regression of a trade shock in period $t+\nu$ on a trade shock in period t, controlling for industry-by-year fixed effects. The coefficient on the current shock z_t is 1 by construction. Standard errors are clustered at the firm level. Panel (b) plots the effect of 100 log point predicted increase in revenue on a firm's log revenue. A predicted 100 log point export demand shock increases revenue by 30 log points on impact, with a peak response of 40 log points the year after the shock. The effect on revenue mostly fades three years after the shock.

3.3 Firm Responses to Export Demand Shocks

Outcome Variables The firm-level outcome variables we consider are log revenue, log employment, the hiring rate, and the wage growth of job switchers and job stayers. We denote revenue by $\mathcal{R}_{j,t}$. Our measure of firm-level employment $N_{j,t}$ is the number of full-time equivalent (FTE) employees. We define the hiring rate as

$$h_{j,t} \equiv \frac{H_{j,t}}{N_{j,t-1}},$$

where $H_{j,t}$ is the number of new hires by firm j in year t. A new hire is defined as a worker who is employed at the firm in November of year t but not in either of the two preceding years. We also require that the new hire worked at least 50 hours at the firm in year t.

A job switcher at firm j at time t is a new hire who was employed at a different firm in the previous two years. We define switcher wage growth for firm j in year t as the average wage growth of the arriving job switchers:

$$\Delta \log w_{j,t}^{switcher} \equiv \frac{1}{|switchers_{j,t}|} \sum_{i \in switchers_{j,t}} (\log w_{i,j,t} - \log w_{i,k,t-2}),$$

where k denotes the prior employer in $t-2$ of the job switcher. We measure wage growth using two-year differences because the annual frequency of the labor market data makes it difficult to know exactly when job transitions are made within a year.

Empirical Specification We estimate the dynamic response to a firm-level export demand shock using reduced-form panel local projections (Jordà, 2005). The export demand shock occurs in year t, and we will measure the response of the firm-level outcome variables from $t-4$ to $t+3$. Our main regression specification is

$$y_{j,t+\nu} - y_{j,t-1} = \beta_\nu^y z_{j,t}^{idiosyncratic} + \delta_\nu^y X_{j,t} + u_{j,t+\nu}^y \tag{14}$$

for each horizon $\nu \in [-4, 3]$, where the long-differenced outcome variable $y_{j,t+\nu} - y_{j,t-1}$ is either log revenue ($\log \mathcal{R}_{j,t+\nu} - \log \mathcal{R}_{j,t-1}$), log employment ($\log N_{j,t+\nu} - \log N_{j,t-1}$), or the hiring rate ($h_{j,t+\nu} - h_{j,t-1}$). Because firms can have zero hires in a year, we leave the hiring variable in levels and convert to elasticities later. The control vector $X_{j,t}$ includes three lags of the export demand shock, the contemporaneous value and three lags of the common shock (as the idiosyncratic and common shocks are negatively correlated mechanically, see Garin and Silvério (2024)), firm fixed effects, industry-by-year fixed effects, and industry-by-year fixed effects interacted with the sum of the lagged export share (Borusyak et al., 2022). We estimate a similar regression for the wage growth of job switchers, except we do not difference the wage growth variable, as the panel of the switcher wage growth is unbalanced and requiring a non-missing value for $\Delta \log w_{j,t-1}^{switcher}$ would greatly reduce sample size. We estimate

$$\Delta \log w_{j,t+\nu}^{switcher} = \beta_\nu^w z_{j,t}^{idiosyncratic} + \delta_\nu^w X_{j,t} + u_{j,t+\nu}^w \tag{15}$$

for each horizon $\nu \in [-4, 3]$. Standard errors are clustered at the firm level.

Results We plot the response of the firm employment level, hiring rate, and wage growth of switchers in Figure 3.[22] Panel (a) shows the response of employment and switcher wage

[22] We align the samples so that the coefficient on firm employment level and switcher wage growth within each time horizon reflects the same sample. However, this means that not every horizon has exactly the same sample, as not every firm makes a full time hire of a switcher each year. The sample for each horizon has between 10,157 and 15,539 firm-year observations.

growth. The solid circles show the effect of this shock on employment, and the hollow triangles report the effect on the wage growth of switchers, where each point represents a different cohort of new hires each year. Conditional on the sequence of shocks from $t-3$ to $t-1$, firms with different values of shocks in period t have similar pre-trends in both the level of employment and in the wage growth of workers who switched into the firm. On impact, an export demand shock that predicts a 100 log point increase in predicted revenue increases a firm's employment level by approximately 15 log points. The effect peaks a year later, with a response of 22 log points. After this, the effect on employment decreases, with employment approximately returning to its pre-shock level three years after the shock. For switcher wages, in the year of the shock, job switchers who move into firms that have a 100 log point export demand shock receive 11 log points higher wage growth than workers who arrive at a firm with a 0 percent export demand shock. However, in the years after the shock, when the firm is larger but is no longer growing, the wage growth of newly hired switchers is similar to or lower than what switcher wage growth was before the shock occurred.

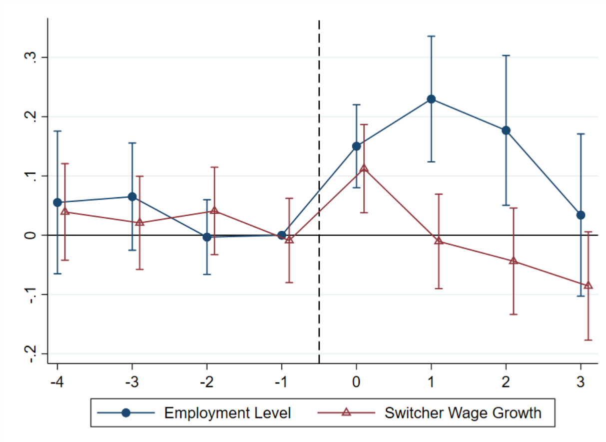
Panel (b) of Figure 3 reports the path of firms' hiring rate and reprises the path of switcher wage growth. On impact, the hiring rate increases by 10 percentage points. Since the average hiring rate is 18.9 percentage points, a 10 percentage point increase corresponds to a 53 percent increase on impact.

Importantly, these figures show that firms pay higher wages to new hires when firms are growing, but not necessarily when firms are large. At $t = 0$, when firms are initially shocked, firms pay high wages when they are growing and rapidly hiring. This is consistent with diminishing returns in hiring ($\chi > 0$). By $t = 1$, firms are larger than before the shock, but the hiring rate and the switcher wage growth are similar to pre-shock levels. These results are also consistent with an elastic labor supply curve ($\sigma = 0$): as long as firms are not rapidly hiring, large firms do not pay higher wages. Our results differ dramatically from static models of monopsony, which would predict that wages would remain above its steady state level as long as the level of the firm's employment is above its steady state.

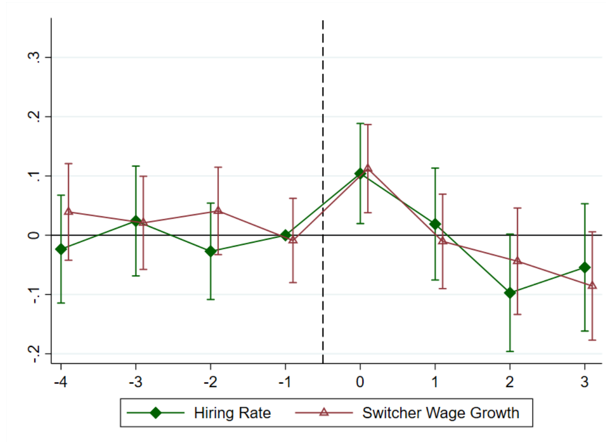
Robustness We now address a number of potential concerns with our empirical strategy. First, the composition of new hires may change in response to the shock. In particular, when hiring accelerates, firms may shift toward younger workers, who typically experience faster wage growth than older workers. Such compositional shifts can mechanically raise estimated wage growth for switchers. In Appendix A, we address this by residualizing switchers' individual wage growth using standard Mincerian controls and then averaging the residuals at the firm–year level. This adjustment has only modest effects, as shown in Figure 6a.

Figure 3: Response of Firm Employment, Hiring Rate, and Switcher Wage Growth to Export Demand Shocks

(a) Employment Level, Switcher Wage Growth



(b) Hiring Rate, Switcher Wage Growth



The figure plots the path of the firm employment level, firm hiring rate, and wage growth of switchers in response to a 100 log point predicted increase in revenue due to an export demand shock in period t. The blue circles in Panel (a) show that firms increase employment by approximately 15 log points on impact, with a peak effect of 22 log points the following year. The hollow red triangles show the effect of the period t trade shock on the wage growth of different cohorts of arriving switchers. Workers who switch into the firm in period t experience wage growth that is 11 log points greater than workers who switch into a firm that is not shocked. However, workers who switch into the shocked firm in subsequent years receive similar or smaller wage increases than workers who arrived prior to the shock. Panel (b) shows that hiring increases by 10 percentage points on impact, while future hiring rates are similar to or below the hiring rates prior to the shock. With an average hiring rate of 18.9 percentage points, a 10 percentage point increase in the hiring rate is equivalent to a 53 percent increase in the hiring rate in the year of the shock.

Second, firms may selectively hire from high- or low-wage firms when expanding. We address this by estimating firm wage effects using the standard two-way fixed effects (AKM) method (Abowd et al., 1999), and include the estimated firm effects when residualizing workers' wages. This adjustment has only very small effects, as shown in Figure 6b in Appendix A.[23]

Third, rapidly expanding firms may relax hiring standards and hire workers who are less qualified than their usual hires. This can bias measured wage changes upward during expansion, conflating a wage premium with occupational upgrading. To address this, we compute market-wide average wages by occupation. Then, when constructing residualized

[23] In Appendix C.2, we allow for heterogeneity in worker types and let firms set wages per efficiency unit. Our model can thus be consistent with firms having time-varying, log-additive wage effects for workers of different types, justifying the inclusion of AKM firm fixed effect controls when residualizing switchers' wages.

wage changes for workers, we include as a control the difference between the average wages of the worker's prior and new occupation. This adjustment has only very small effects, as shown in Figure 6c in Appendix A.

Fourth, an idiosyncratic shock to a large firm may affect the local labor market, violating the Stable Unit Treatment Value Assumption (SUTVA)—no interference across units and no multiple versions of treatment. In general, we are not concerned that this materially affects our results: the standard deviation of the shock is 0.046 log points, implying approximately a 1 percent increase in employment in the second year. The median firm in our sample has fewer than 40 employees, so a typical shock generates fewer than one additional full-time-equivalent employee after two years. This is a very small shock in the local labor market, so we assume SUTVA holds. However, as a robustness check, we estimate responses of employment and wages for small firms with log full-time-equivalent employment less than four (approximately 55 full-time-equivalent workers) and report the results in Figure 6d in Appendix A. We find broadly similar results: employment rises on impact and remains elevated for a few periods, while switchers' wage growth increases on impact but declines once the firm is no longer expanding.

Lastly, firms may be hiring from other firms that are also affected by the shock, particularly when typical hires come from competing firms or similar industries. If the shock does not perfectly isolate idiosyncratic demand shifts, expanding firms may hire from similarly affected firms. To address this, we reconstruct the wage-growth measure using only hires from non-exporting firms and report the paths of employment, hiring, and switcher wage growth in Figure 6e and 6f. We find a nearly identical pattern: switchers' wage growth rises and falls with the hiring rate, while employment remains elevated even after switchers' wage growth returns to pre-shock levels.

Stacked Regression Next we estimate a truncated version of the impulse responses in a stacked regression. We use these estimates in the next subsection when we estimate our structural parameters χ and σ, residualizing for worker demographics as described in the first robustness check in this section. Specifically, we stack the dependent variables in equations (14) and (15) and estimate the regression jointly. This imposes that all variables are non-missing, which particularly binds for the switcher wage growth variable, shrinking the sample size. The results are reported in Table 4. The estimates are similar to those in Figure 3.

Table 4: Estimated Coefficients from Stacked Regression

$\hat{\beta}_0^w$	$\hat{\beta}_1^w$	$\hat{\beta}_2^w$	$\hat{\beta}_0^h$	$\hat{\beta}_1^h$	$\hat{\beta}_2^h$	$\hat{\beta}_0^N$	$\hat{\beta}_1^N$	$\hat{\beta}_2^N$
0.141	0.016	-0.002	0.146	0.025	-0.042	0.158	0.251	0.176
Observations: 8,384								

This table reports the estimated coefficients from a stacked regression of equations (14) and (15) for horizons $\nu \in [0, 2]$, which jointly estimate the responses of switchers' wage growth, the hiring rate, and firms' log employment level to idiosyncratic export demand shocks.

3.4 Estimating the Recruitment Cost Function Parameters

In the previous section, we estimated the causal effect of an idiosyncratic export demand shock on the path of firm employment levels, hiring rates, and switcher wage growth. In this section, we use the results to estimate our model parameters of interest χ and σ and to obtain estimates for the inverse labor supply elasticity and profit share of marginal product.

We estimate the recruitment parameters χ and σ using minimum distance estimation. Our estimation strategy involves plugging empirical estimates into a model-derived equation that describes how the variables are related, and then choosing parameter values to minimize the sum of squared errors of that expression.[24] Define π as the sequence of the firm's log linearized choice variables: $\pi \equiv [\tilde{w}_t \;\; \tilde{w}_{t+1} \;\; ... \;\; \tilde{h}_t \;\; \tilde{h}_{t+1} \;\; ... \;\; \tilde{N}_t \;\; \tilde{N}_{t+1} \;\; ...]'$. These variables are the model objects that describe the response of the firm's choice variables to a product demand shock in period t in terms percentage deviations from steady state. The empirical analogue $\hat{\pi} \equiv [\hat{\beta}_0^w \;\; \hat{\beta}_1^w \;\; ... \;\; \hat{\beta}_0^h/\bar{h} \;\; \hat{\beta}_1^h/\bar{h} \;\; ... \;\; \hat{\beta}_0^N \;\; \hat{\beta}_1^N \;\; ...]$ is the estimated response of the corresponding variables in the data to an identified idiosyncratic demand shock (scaling the hiring rate by $\bar{h}$ to express it in percentage deviations).

Equation (13) describes how the endogenous variables are related to each other in response to a shock in the model. This equation provides infinitely many conditions, as w_t depends on N_{t-1}, period t variables, and future variables; w_{t+1} depends on N_t, period $t+1$ variables, and future variables, and so on. We truncate this infinite sequence of equations to T conditions and express them as

$$f(\pi, \chi, \sigma, \Theta) \equiv \mathcal{A}(\sigma, \chi, \Theta)\pi = 0, \tag{16}$$

where $\mathcal{A}(\sigma, \chi, \Theta)$ is a matrix with elements composed of σ, χ, and calibrated model objects $\Theta \equiv \{S, \gamma\}$ that turns the conditions of equation (13) into a T equation linear system. Using

[24] Estimating the parameters using standard indirect inference would be challenging, as the large state variable of past committed wages for each cohort of workers makes simulating the model difficult.

$\gamma = 4$ and a steady-state separation rate $S = .05$, we plug in for π with $\hat{\pi}$ from Table 4 and choose σ and χ to minimize a quadratic loss function

$$\min_{\sigma,\chi} f(\hat{\pi}, \sigma, \chi, \Theta)' f(\hat{\pi}, \sigma, \chi, \Theta). \tag{17}$$

The parameters are just identified if $T = 2$.[25] We provide detailed steps in Appendix A.1.

One technical issue is that our model is best calibrated at a monthly frequency (to limit time aggregation bias), while our empirical results are annual. We discuss how we map our annual empirical estimates into the monthly model in Appendix A.1. Broadly speaking, we assume that hires are evenly distributed within a year, and we plug in each of our annual estimates of $\hat{\pi}$ into 12 periods of corresponding model "months". We use the first two years of estimated switcher wages and truncate the infinite sum in equation (13) such that switcher wages depend on future variables at most one year in advance. Converting to a monthly linear system gives 24 "months" of equations that depend on 9 estimated annual variables in $\hat{\pi}$, making $\mathcal{A}(\sigma, \chi, \Theta)$ a 24×9 matrix. $\mathcal{A}(\sigma, \chi, \Theta)$ is thus constructed such that the wages of a worker hired earlier within a year (suppose year 0) have more weight on $\hat{\beta}_0^h$ and less weight on $\hat{\beta}_1^h$ than does a worker hired later in year 0.

Table 5: Fitted Parameters χ and σ and Inverse Labor Supply Elasticity $\varepsilon_{w,N}$

Parameter	χ	σ	$\varepsilon_{w,N}$
Estimate	2.21	0.009	0.001
10th percentile	0.79	-2.63	-0.18
90th percentile	3.95	3.33	0.27

This table reports point estimates and the 10th/90th percentiles for χ, σ, and the implied inverse labor supply elasticity $\varepsilon_{w,N}$ obtained via minimum-distance estimation.

Table 5 reports the point estimates for σ, χ, and the inverse labor supply elasticity $\varepsilon_{w,N}$. We estimate $\sigma = 0.009$, $\chi = 2.21$, and the inverse labor supply elasticity $\varepsilon_{w,N} = 0.001$. To characterize the uncertainty of our estimates, we construct a distribution of estimates by sampling from the variance-covariance matrix of coefficients, fitting σ and χ to minimize (17) for each draw. The 10th and 90th percentiles of estimates for χ are 0.79 and 3.95, respectively. The 10th and 90th percentiles of estimates for the inverse labor supply elasticity are -0.18 and 0.27, respectively. We report distributions of the estimates in Appendix A.1.[26]

[25] Intuitively, identification of the χ and σ come from different timing of the responses of hiring and employment. χ is primarily influenced by the co-movement of wages with the hiring rate, and identification of σ comes from residual variation of wages with employment that hiring cannot explain.

[26] We report percentiles and distributions because our estimates of $\hat{\chi}$ are not normally distributed.

3.5 Worker-Level Regressions for Switchers and Stayers

In the previous section, we focused on the path of new hire wages across arrival cohorts of workers. Three natural questions arise. First, are workers who switch into expanding firms on different wage trajectories? Second, are the wage effects for new hires persistent after moving firms? Third, how do the wages of job stayers respond?

To answer these questions, we estimate the path of individual worker wages for both new hires and stayers in an event study specification. First, we find that workers who switch into positively shocked firms do not have significantly different pre-trends than comparable workers who switch into firms that receive a zero shock. Second, wage effects for workers hired in the shock year are persistent, supporting our modeling assumption that firms commit to posted wages, while workers hired the following year show no effect. Third, stayers' wages increase modestly in response to the export demand shocks, consistent with the literature's estimates of firm-specific labor supply elasticities of around 4 when estimated using stayer wages.

Job Switchers In this section, we estimate an event study showing the path of individual workers' wages as they change jobs, using worker-level local projections. As before, we focus on full-time workers who also worked full-time in their previous job. Therefore, job switchers are defined as workers in their "main job" who worked full-time (at least 1,400 hours) at firm j in year t and were full-time in their "main job" at some firm $k \neq j$ in year $t-2$. We use two-year changes because of how the data is constructed: employment is reported as of a worker's employer in November. Workers who are full-time in year t may have switched jobs in year $t-1$, but would not yet have completed a full year with the new employer by November of year $t-1$.

Equation (18) estimates the effect of a period t shock on a worker's wage growth from $t-2$ to $t+\nu$. The sequence $\{\beta^\nu\}$ traces out the effect of an export demand shock hitting in year t on the path of wages for workers who switch into the firm in year t. Define $\Delta \log w_{i,t+\nu} \equiv \log w_{i,t+\nu} - \log w_{i,t-2}$.

$$\text{Arriving in year of shock:} \quad \Delta \log w_{i,t+\nu} = \beta^\nu z_{j(i,t),t} + \zeta x_{i,t} + \delta^\nu X_{j(i,t),t} + u_{i,t+\nu}, \tag{18}$$

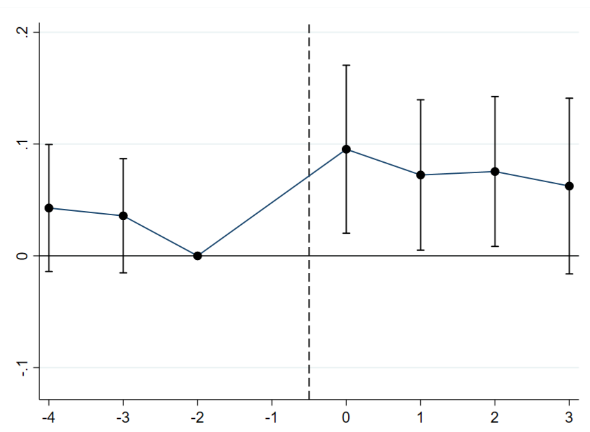
$$\text{Arriving year after shock:} \quad \Delta \log w_{i,t+\nu} = \tilde{\beta}^\nu z_{j(i,t),t-1} + \tilde{\zeta} x_{i,t} + \tilde{\delta}^\nu X_{j(i,t),t} + \tilde{u}_{i,t+\nu}, \tag{19}$$

where $j(i,t)$ is the firm that worker i is employed at in year t, $z_{j(i,t),t}$ is the firm-level export demand shock, and controls $x_{i,t}$ include gender, years of education, potential experience, its square, and the interaction of potential experience with education, and $X_{j(i,t),t}$ includes firm fixed effects, industry-by-year fixed effects, industry-by-year fixed effects interacted with firm's export share of revenue in $t-1$, three years of the lagged idiosyncratic shock, and the contemporaneous and three years of lags of the common shock. Hence, (18) addresses the question: How does an export demand shock in year t affect the wages of job switchers that arrive at the firm in year t?

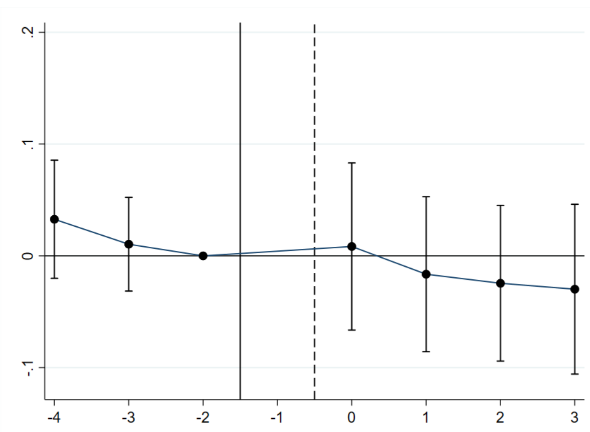
As an additional test, we also estimate (19), which is the same regression but excludes the time t shock, and the controls include industry-by-year fixed effects and industry-by-year effects interacted with the export share of revenue in year $t-2$. That way we can get a consistent estimate of the coefficient β_{-1}, which estimates the effect of an export demand shock in period $t-1$ on the path of wages for workers who arrive in year t. Controls of lagged shocks include shocks through year $t-4$.

Figure 4: Path of Switcher Wage Level by Trade Shock Timing Relative to Job Switch

(a) Workers Arrive in Trade Shock Year



(b) Workers Arrive in Year After Trade Shock



This figure reports the estimated coefficients from equations (18) and (19). Panel (a) plots the path of an individual worker's wage level for a worker who switches in year t into a firm that experiences a 100 log-point positive shock to revenue in year t. Panel (b) plots the path of an individual worker's wage level for a worker who switches in year t into a firm that experiences a shock of the same size in year $t-1$.

Figure 4 shows the estimates from equations (18) and (19). Figure 4(a) shows the coefficient of the time t shock on the wage growth of workers from $t-2$ to $t+\nu$. Visual inspection of pre-trends indicates that workers moving into differently shocked firms have similar wage paths prior to switching. Consistent with evidence in Figure 3a, wages of new hires are approximately 10% higher at firms facing a 100 log-point shock to predicted revenue relative to workers who arrive at firms with a zero shock. The wage premium for workers hired in the shock year is persistent.

Figure 4(b) reports the coefficient on the lagged shock, $z_{j(i,t),t-1}$, on the wage changes of switchers who arrive in period t from a local-projection specification analogous to equation (18), omitting the trade shock in period t.[27] This figure shows that workers who arrive in firms that had larger export demand shocks in period $t-1$ receive no wage premium upon arrival in period t and exhibit no subsequent catch-up in wage growth. This provides further evidence that firms do not need to pay new hires higher wages once the firm has already expanded.

Job Stayers In addition to estimating the path of wages for switchers around export demand shocks, we also estimate the wage growth of job stayers. We define a stayer as a worker who was full-time ($\geq$ 1,400 hours/year) at the same firm from in years $t-1$ through $t+\nu$. We estimate a local projection of the change in workers' wages relative to year $t-1$, with the firm export-demand shock in year t as the independent variable. We regress

$$\log w_{i,t+\nu} - \log w_{i,t-1} = \bar{\beta}^{\nu} z_{j(i,t),t} + \bar{\delta}^{\nu} X_{i,t} + \tilde{u}_{i,t+\nu} \tag{20}$$

for $\nu \in [-4, 3]$, where $X_{i,t}$ includes firm fixed effects, three lags of the idiosyncratic shock $z_{j(i,t),t}$, the current value and three lags of the common shock, industry-by-year fixed effects, and industry-by-year fixed effects interacted with $t-1$ export shares. Standard errors are clustered at the firm level.

Table 6: Response of Stayer Wages to Export Demand Shock

	(1)	(2)	(3)
VARIABLES	$\Delta \log w_{i,t-1,t}$	$\Delta \log w_{i,t-1,t+1}$	$\Delta \log w_{i,t-1,t+2}$
Trade Shock $z_{j(i,t),t}$	0.0159	0.0521**	0.0290
	(0.0274)	(0.0219)	(0.0245)
Observations	1,749,961	1,355,183	1,059,102

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Notes: Coefficients from the worker-level local projection in (20). The dependent variable is $\log w_{i,t+\nu} - \log w_{i,t-1}$. The regressor $z_{j(i,t),t}$ is the firm-level idiosyncratic export-demand shock. Sample: full-time stayers ($\geq$1,400 hours/year) who remain at the same firm from $t-1$ through $t+\nu$. Controls $X_{i,t}$ include three lags of the idiosyncratic shock, the current value and three lags of the common shock, industry-by-year fixed effects, and their interactions with $t-1$ export shares. Standard errors clustered at the firm level. Coefficients are scaled to a 100 log-point shock in predicted revenue; entries are in log points.

Table 6 reports the response of stayer wages to an export-demand shock. The peak

[27] We correspondingly include the industry-by-year fixed effects interacted with export shares for period $t-2$ instead of for year $t-1$.

response to a 100 log-point predicted increase in revenue is the year after the shock, when stayer wages rise by 5.2 log points. As a back-of-the-envelope calculation, the implied inverse labor supply elasticity estimated using the response of stayer wages one year after the shock is $d \log w^{\text{stayers}}/d \log N = (d \log w^{\text{stayers}}/d \log Y)/(d \log N/d \log Y) = 0.052/0.22 = 0.236$, so the implied labor supply elasticity is $1/0.236 \approx 4.2$. Thus, we replicate the literature's finding that using stayer wages to calculate labor supply elasticities yields a value of around 4, implying much less elastic labor-supply curves than the (effectively) perfectly elastic long-run labor-supply curves we find using switchers in Section 3.4. Since firms' long-run marginal cost of labor is determined by the wages of new hires, the wages of new hires determine markdowns in steady state, not the wages of stayers. Accordingly, using the pass-through of demand shocks to stayer wages will understate firms' labor supply elasticity.

3.6 The Firm Size Wage Premium

So far, we have shown that when manufacturing firms are subject to export demand shocks, these firms do not pay any higher wages to new hires than the firms did prior to the shock. In this section, we estimate the the effect of firm size on workers' wages for Danish firms economy-wide between 2008-2019. We regress workers' log wages on the log of a firm's average employment level in full-time equivalent (FTE) workers, averaged over 2008-2019. We do this for two separate samples of firms: first restricting to firms with 5 or more full-time equivalent workers, and second, restricting to firms with 25 or more FTEs:

$$\log w_{ijt} = \alpha_i^{\bar{N}} + \beta^{\bar{N}} \overline{\log N_j} + \xi_{j,t}^{\bar{N}} + e_{ijt}^{\bar{N}},$$

where $\alpha_i^{\bar{N}}$ is a worker fixed effect, $\xi_{j,t}^{\bar{N}}$ is a firm-level industry-by-year fixed effect. We cluster the standard errors at the firm level.

Table 7 reports the results. For firms with 5 or more workers, the elasticity of workers' wages with respect to firm size is 0.005, or 0.5%. Focusing only on firms with on average 25 workers or more, the elasticity drops to .001, or around 0.1%. These estimates are similar to the elasticities of firm wage effects on log firm size estimated in the United States (Bloom et al., 2018), particularly for medium-sized and large firms, suggesting that our results are not solely a feature of the high union density in Denmark.

Table 7: Firm Size Wage Premium

	(1)	(2)
VARIABLES	$\log w_{ijt}$	$\log w_{ijt}$
$\overline{\log N_j}$	0.00531***	0.00146**
	(0.000543)	(0.000706)
	$\exp(\overline{\log N_j}) \geq 5$	$\exp(\overline{\log N_j}) \geq 25$
Observations	17,389,575	14,475,115
No. Firms	53200	9885

*** p<0.01, ** p<0.05, * p<0.1

This table reports regressions estimating the effect of firm size on workers' log wages that includes worker fixed effects and industry-by-year fixed effects. For firms with an average employment of at least 5 full time equivalent workers, the elasticity of worker wages to firm size is 0.005. For firms with at least 25 workers, the elasticity of wages to firm size is 0.001. Standard errors are clustered at the firm level.

4 Profit Puzzle with Oligopsonistic Competition

In this section, we extend the equilibrium analysis in Section 2 to allow for an oligopsonistic labor market with strategic behavior. Using our evidence for the recruiting cost parameters from Section 3, we characterize the aggregate income shares for labor, capital, and profits as a function of parameters and the level of labor market concentration. We show that in a calibration that shuts down the recruiting margin and assumes a labor-supply elasticity of 4, the implied labor share is at least 10 percentage points too low when price markups and capital income are calibrated to realistic levels. We then show that with constant-returns-to-scale recruiting function and a long-run elastic labor supply curve, the model matches a labor share in line with national accounts data in rich countries. Furthermore, we find that moving from an equilibrium with no labor market concentration to an empirically realistic level of concentration (an employment Herfindahl–Hirschman Index (HHI) of 0.1) reduces wages relative to marginal product by only 3.4%, an order of magnitude smaller than what is implied by models with Cournot competition and no recruiting margin (Berger et al., 2022).

Setting We now enrich the model from Section 2 by assuming that firms are non-atomistic and internalize that their actions affect labor market aggregates, as well as that workers may encounter their own firm's vacancies when searching. We restrict to no exogenous separations ($s_0 = 0$). Define $\phi^n_{j,s,t-1}$ as the share of employed workers who were hired in period s by firm j and are still employed by firm j in period $t-1$, define $\phi^n_{j,t} \equiv \frac{N_{j,t}}{\sum_k N_{k,t}}$ as firm j's share of employment in period t with $\phi^n_{k,t-1} = \sum_{s=-\infty}^{-1} \phi^n_{k,s,t-1}$, and define $\phi^v_{j,t} = \frac{V_{j,t}}{\sum_k V_{k,t}}$ as firm j's share of vacancies posted in period t. We assume that if a worker encounters their own firm's vacancy, the worker redraws preferences and chooses between the two jobs as if they were two different firms but keeps their original cohort s's wage. From the perspective of the firm, there is no net change in employment, and thus the number of separations or recruits is unaffected. The recruiting and cohort-specific separation functions take the form

$$R\big(\mathcal{V}_{j,t,t}, \mathcal{V}_{-j,t}, V_{j,t}, V_{-j,t}, N_{j,t-1}, N_{-j,t-1}\big) = \\ g(\theta_t)\Big[(1-\Phi^U_{t-1})\sum_{k\neq j}\sum_{s=-\infty}^{t-1} \phi^n_{k,s,t-1} \frac{\exp\big(\gamma\,\mathcal{V}_{j,t,t}\big)}{\exp\big(\gamma\,\mathcal{V}_{j,t,t}\big)+\exp\big(\gamma\,\mathcal{V}_{k,s,t}\big)} + \Phi^U_{t-1}\,\mathbb{1}\{\mathcal{V}_{j,t,t} > \mathcal{V}_U\}\Big] \tag{21}$$

$$S\big(\mathcal{V}_{j,s,t}, \mathcal{V}_{-j,t}, V_{j,t}, V_{-j,t}\big) = \\ \lambda_{EE} f(\theta_t)\sum_{k\neq j} \phi^v_{k,t} \frac{\exp\big(\gamma\,\mathcal{V}_{k,t,t}\big)}{\exp\big(\gamma\,\mathcal{V}_{j,s,t}\big)+\exp\big(\gamma\,\mathcal{V}_{k,t,t}\big)} + \big(1-\lambda_{EE} f(\theta_t)\big)\,\mathbb{1}\{\mathcal{V}_U > \mathcal{V}_{j,s,t}\} \tag{22}$$

where $\mathcal{V}_{-j,t}$ denotes the value of all workers not at firm j, $\Phi^U_{t-1} \equiv \frac{U_{t-1}}{U_{t-1}+\lambda_{EE}(1-U_{t-1})}$, $\theta_t \equiv \frac{\sum_k V_{k,t}}{U_{t-1}+\lambda_{EE}(1-U_{t-1})}$ denotes labor-market tightness, and $U_{t-1} = 1 - \sum_k N_{k,t-1}$. We assume that firms operate in an extended version of Bertrand competition, where firms solve their problems taking as given the wage and vacancy policies of their competitors.

Firm's Problem The firm's problem differs from the firm's problem in Section 2 in two ways. First, the firm knows that its own workers will not quit if the worker encounters their own firm's vacancy. Second, the firm internalizes that their choice of vacancies affects labor market tightness, and that additional vacancies congest the firm's own recruiting, as well as congest the labor market for the firm's current workers. Firms solve (23) taking $\{V_{-j,t}\}_{t=0}^{\infty}$ and $\{w_{-j,t}\}_{t=0}^{\infty}$ as given.

$$\max_{\left\{V_{j,t},\, K_{j,t},\, N_{j,t},\, \{N_{j,s,t}\}_{s\leq t},\, \{w_{j,s,t}\}_{s\leq t}\right\}_{t=0}^{\infty}} \\ \sum_{t=0}^{\infty}\beta^t\left[A_{j,t}\left(K^{\alpha}_{j,t}N^{1-\alpha}_{j,t}\right)^{\frac{\epsilon-1}{\epsilon}} - \sum_{s\leq t} w_{j,s,t}\,N_{j,s,t} - r^K K_{j,t} - c_j\left(\frac{V_{j,t}}{N_{j,t-1}}\right)^{\chi} N^{\sigma}_{j,t-1}V_{j,t}\right] \tag{23}$$

subject to (6), (7), (8), and (9), where the recruiting and separation functions are (21) and (22), and labor-market clearing condition is $U_t + \sum_k N_{k,t} = 1$, and tightness is $\theta_t = \frac{\sum_k V_{k,t}}{U_{t-1}+\lambda_{EE}(1-U_{t-1})}$. As before, we will focus on the limiting case where $\beta_f \to 1$.

Equilibrium A stationary equilibrium is defined as in Section 2, except that each firm maximizes profits, taking competitors' vacancy and wage strategies as given.

Markdowns We now characterize recruiting cost-adjusted markdowns for the economy with oligopsony in the labor market. We can set b sufficiently low so that workers never voluntarily quit into unemployment, implying zero unemployment in equilibrium. Since all workers are employed in equilibrium, and under the normalization $\sum_k N_{k,t} = 1$, firm j's share of employment is equal to its employment level, $\phi^n_{j,t} = N_{j,t}$.

We now highlight a special case where all firms choose the same wage in equilibrium, which allows us to characterize markdowns in closed form.

Proposition 2. *If the unemployment benefit b is sufficiently low, workers fully discount ($\beta_w \to 0$), firms have common recruiting cost shifters $c_j = c \; \forall j$, and if either $\sigma = 0$ or firms are ex-ante identical ($A_j = A \; \forall j$), then all firms within a market pay the same wage.*

If in addition workers have log utility ($\eta \to 1$), then the recruiting cost-adjusted markdown takes the form

$$\mu_j \equiv \frac{w_j + c\left(\frac{V_j}{N_j}\right)^{1+\chi} N_j^\sigma}{MRPL_j} = \frac{1+\gamma(1+\chi)}{1+\gamma(1+\chi)+\sigma+(1+\chi)\frac{\phi^n_j}{1-\phi^n_j}}. \tag{24}$$

Proof: See Appendix B.6. Intuitively, for the first claim about equal wages, two forces offset exactly when $\sigma = 0$: (i) for larger firms, the marginal recruiting gain from a higher wage is lower because a greater share of searchers they meet are already employed by the firm (no competition from one's own vacancies); (ii) the marginal value of an additional worker is higher because larger firms strategically under-post vacancies, raising marginal product. The offset implies that small and large firms pay identical wages, consistent with the lack of a firm-size wage premium in Section 3.6. This is shown by equation (56) in Appendix B.6.

Labor and Profit Shares in Equilibrium Now we characterize the share of income that goes to labor, capital, and profits. We will assume that turnover costs accrue to labor income.[28] In equilibrium, the share of income that goes to wage income and turnover costs (which are allocated to wages) as a share of gross output is

$$\text{Gross Labor Share} \equiv \sum_j \frac{w_j N_j + c\left(\frac{V_j}{N_j}\right)^{1+\chi} N_j^\sigma}{P_j Y_j} = \frac{\epsilon-1}{\epsilon}(1-\alpha)\frac{1}{\sum_j(\phi^n_j/\mu_j)}.$$

We can define the aggregate wage markdown as $\mathcal{M} = 1/(\sum_j(\phi^n_j/\mu_j)$, and the Herfindahl-Hirschman index is $\sum_j(\phi^n_j)^2$. As we will discuss momentarily, we will be interested in the

[28]As before, our results are not sensitive to this choice. For more details, see Appendix B.4.

labor share of net value added, which subtracts depreciation of capital from GDP. Subtracting out depreciation from the denominator, the net labor share and profit share are

$$\text{Net Labor Share} = \frac{\frac{\epsilon-1}{\epsilon}(1-\alpha)\mathcal{M}}{1-\frac{\epsilon-1}{\epsilon}\alpha\frac{\delta}{\delta+r^K}}, \qquad \text{Net Profit Share} = \frac{\frac{1}{\epsilon}+\frac{\epsilon-1}{\epsilon}(1-\alpha)(1-\mathcal{M})}{1-\frac{\epsilon-1}{\epsilon}\alpha\frac{\delta}{\delta+r^K}},$$

where the profit share of gross value added from product market power is $\frac{1}{\epsilon}$, and the profit share of gross value added from wage markdowns is $\frac{\epsilon-1}{\epsilon}(1-\alpha)(1-\mathcal{M})$.

Evidence on Profit and Labor Shares The labor share measure that we focus on is the net labor share of domestic corporate value added, which excludes housing and proprietor's income. Our preferred measure of the labor share excludes housing because housing is not traditionally a factor of production, and focusing on the corporate share allows us to avoid taking a stance on how to allocate proprietor's income, which can be challenging (Elsby et al., 2013). The "net" part of net domestic corporate value added means that depreciation is subtracted from value added, since payments to capital that pay off depreciation are not available for consumption (Koh et al., 2020). Rognlie (2015) shows for advanced economies, the labor share of net value added for the domestic corporate sector is between 70-80%.[29] The allocation of non-labor income between capital income and pure profit is usually not measured directly, so we will use additional evidence and the model to infer a capital and profit share of income.

Calibration and Labor Share We now calibrate our model and compute the labor and profit shares of income under three scenarios: (i) a standard atomistic monopsony case, (ii) our preferred calibration with a constant returns to scale (CRS) recruiting function, and (iii) an extension with labor market concentration.

Table 8: Calibration

Parameter	Description	Value	Rationale
ϵ	Demand elasticity	7	Price markup
r^K	Rental rate of capital	0.125	Return on capital = 0.05
δ	Depreciation rate	0.075	Match depreciation/GDP
α	Cobb–Douglas capital elasticity	0.3	Match $K/Y \approx 2$
γ	Inverse scale parameter of non-wage preferences	4	Match $\varepsilon_R - \varepsilon_S = 4$
s_0	Exogenous separation rate	0	Tractability

[29]For the United States, data on value added of nonfinancial domestic corporate business can be found in Table 1.15 of the National Income and Product Accounts.

Table 8 presents our choice of parameter values that are constant across our three scenarios. We set the rate of exogenous separations at $s_0 = 0$, so $\varepsilon_{R,w} - \varepsilon_{S,w} = \gamma$ at any wage level, and we set the inverse scale parameter of workers' preferences over non-wage amenities $\gamma = 4$. We set the product demand elasticity $\epsilon = 7$ to generate a price markup over marginal cost of 16.7%, which is relatively small but consistent with empirical evidence (Kline et al., 2019). We assume an ad hoc risk premium on capital of $r = 0.05$ (Jordà et al., 2019), a risk-free real rate of zero, and a depreciation rate of $\delta = 0.075$. This yields a rental rate of capital $r^K = 0.125$. Together with a Cobb–Douglas capital share $\alpha = 0.3$, the parameters jointly deliver a ratio of capital to output $K/Y \approx 2$, consistent with Rognlie (2018), a capital-depreciation share of GDP of 15%,[30] and a gross capital share of income of 25%, consistent with Barkai (2020). As can be seen in the adjusted markdown expression in equation (24), all other parameters that govern the search aspect of the model drop out.[31]

Table 9 reports the share of income that goes to labor, capital, and profits under three scenarios. In the first column, we return to the assumption of atomistic firms, so $\phi_j^n = 0$ for all firms, but the recruiting margin is shut down ($\sigma = \chi \to \infty$). In this case, wages are 20% below marginal product. Profits from price markups over marginal cost account for 17% of net value added, and net capital income (excluding depreciation) accounts for 12% of net value added. In total, this leaves a profit share of net value added equal to 31%, while the labor share of net value added is only 57%, significantly below the empirical range of 70-80%.

In the second column, we assume a constant returns to scale recruiting function ($\sigma = 0$) with $\chi = 2$ and only atomistic firms (HHI = 0). Product market profits and net capital shares are the same as in column 1, so the only difference is that the labor share of net value added is 71%, i.e., 14 percentage points higher than in the no recruiting case. As a consequence, the profit share of net value added is 14 percentage points lower. This case matches national accounts data on labor and non-labor shares of national income in rich countries.

In the third column, we assume there is one large firm that accounts for 31.6% of employment, yielding a labor market HHI of 0.1.[32] Compared to the case with no concentration, the recruiting cost-adjusted markdown is 3.4% lower, lowering the aggregate labor share of net income by 2 percentage points, putting the labor share just outside the empirical range.

[30] See National Income and Product Accounts (NIPA) Table 1.11.

[31] For example, the on-the-job search probability λ_{EE} and the cost of posting a vacancy c do not matter for markdowns and consequently labor and profit shares. Regardless, we set $\lambda_{EE} = .14$ to get a monthly job-to-job separation rate just above 2% (2.3% in (Bagger et al., 2022)) and $c = 1024$ to generate a job openings rate of 4.3% (4.1% in Bagger et al. (2022)).

[32] We show in Appendix B.7 that for a given HHI, the widest markdown is achieved by concentrating employment in only one firm.

For labor markets that are less concentrated, as found by Schubert et al. (2023) (median HHI of 0.018 on a 0–1 scale), the labor share of net value added would rise back above 70%.

Table 9: Distribution of Income: Standard Monopsony, CRS Recruiting, and Concentration

	Standard Monopsony $\sigma = \chi,\ \sigma, \chi \to \infty$	CRS Recruiting $\sigma = 0,\ \chi = 2$	CRS + Oligopsony HHI = 0.1	National Accounts Data
recruiting cost-adjusted Markdown	0.80	1.00	0.97	
Product Profits/Net Output	0.17	0.17	0.17	
Total Profit/Net Output	0.31	0.17	0.19	
Net Capital Income/Net Output	0.12	0.12	0.12	
Non Labor Income/Net Output	0.43	0.29	0.31	0.20–0.30
Labor Income/Net Output	0.57	0.71	0.69	0.70–0.80

This table reports the recruiting cost-adjusted markdown and the shares of aggregate net value added: product market profits, total profit, net capital income, non labor income, and labor income. For each model, we assume the labor supply elasticity conditional on vacancies $\mathcal{E} \equiv \varepsilon_{R,w} - \varepsilon_{S,w} = 4$. The three economies are (i) a standard monopsony model with no recruiting margin; (ii) our dynamic monopsony model with a constant returns to scale recruiting function; and (iii) one large firm with employment share $\sqrt{0.1} \approx 0.316$ (yielding labor market HHI = 0.1) while all other firms are small.

Could the puzzle be resolved by eliminating price markups? As shown in Column 1 of Table 9, profits from pricing power in the product market as a share of net value added (17%) are roughly equal to profits from wage setting power in the labor market (31% − 17% = 14%). What happens to the profit share if we shut down product market rents? To calculate this, we recalibrate the capital elasticity to $\alpha = 0.25$ to get a capital to output ratio of 2. Holding other parameters fixed with $\gamma = 4$, $\sigma = \chi$, $\sigma, \chi \to \infty$ (no recruiting margin), and letting $\epsilon \to \infty$ (no price markups), we obtain a net labor share of 71%. Thus, under a labor supply elasticity of 4 and no recruiting margin, matching the observed labor share of net output requires eliminating price markups.

Misallocation We now consider misallocation. Because large firms strategically under-hire, there may be dispersion in marginal products.[33] Using our result on markdowns from equation (24), and using the calibration with $\sigma = 0$ and $\chi = 2$, we numerically assess the output losses from misallocation due to large firms exercising power in the labor market. We reverse engineer firm-specific demand shifters that ensure that the large firm employs a share 31.6% of employment in the market, so the labor market HHI is 0.1. We then compute

[33] Even without oligopsony, search frictions can generate inefficiency. We do not consider distance to the planner's allocation.

the steady-state local market output $Y_m = \left(\sum_j Y_{j,m}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$ and the capital consumption $\delta K_m = \delta \sum_j K_{j,m}$. We then reallocate labor within the local labor market to equalize marginal products and calculate the reallocated output Y_m^* as well as capital consumption δK_m^* given re-optimized choices for capital. The net output gain from reallocating labor is $(Y_m^* - \delta K_m^*)/(Y_m - \delta K_m) = 1.006$, so output increases by 0.6%. At comparable levels of labor market concentration (HHI), our estimated misallocation-induced output losses are about an order of magnitude smaller than those reported by Berger et al. (2022).

5 Conclusion

In this article, we provide a model of dynamic monopsony where workers have heterogeneous preferences over firms and can search on the job, and firms can attract workers with higher wages and recruiting expenditures. We use this model to analytically decompose the share of marginal product into wages, recruiting costs, and profits. We provide empirical evidence consistent with constant returns to scale recruiting functions, implying that firms' long-run labor supply curves are very elastic, and that the rents that firms get from exploiting their wage setting power are spent on recruiting. We show that, in equilibrium, elastic labor supply curves help match aggregate profit, capital, and labor shares of income if firms also have pricing power in the product market, even taking into account labor market concentration and strategic interactions. Our setting tractably combines numerous sources of monopsony, including preference heterogeneity, search frictions, and labor market concentration, in a way that is easily disciplined by new and existing empirical evidence and nests many existing monopsony models.

The tractability of our model may be useful for researchers who wish to model a labor market with on-the-job search but do not want the added complexity of an equilibrium wage distribution. For example, Bloesch et al. (2024) use a similar setting in the labor block of a New Keynesian model to study how wages evolve in response to cost-of-living shocks if firms post wages and workers search on the job. In general, our model may be useful for tractably modeling other search environments such as spending on advertising, where buyers or sellers remain matched while searching for a new match.

References

Abowd, J. M., F. Kramarz, and D. N. Margolis (1999). High Wage Workers and High Wage Firms. *Econometrica 67*(2), 251–333.

Acemoglu, D. and W. B. Hawkins (2014). Search with Multi-Worker Firms. *Theoretical Economics 9*(3), 583–628.

Albrecht, J., C. Carrillo-Tudela, and S. Vroman (2018). On-the-Job Search with Match-Specific Amenities. *Economics Letters 162*, 15–17.

Bagger, J., F. Fontaine, M. Galenianos, and I. Trapeznikova (2022). Vacancies, Employment Outcomes and Firm Growth: Evidence from Denmark. *Labour Economics 75*, 102103.

Barkai, S. (2020). Declining Labor and Capital Shares. *The Journal of Finance 75*(5), 2421–2463.

Bassier, I., A. Dube, and S. Naidu (2022). Monopsony in Movers The Elasticity of Labor Supply to Firm Wage Policies. *Journal of Human Resources 57*(S), S50–s86.

Berger, D., K. Herkenhoff, and S. Mongey (2022). Labor Market Power. *American Economic Review 112*(4), 1147–1193.

Bertheau, A., P. Cahuc, S. Jager, and R. Vejlin (2021). Turnover Costs: Evidence from Unexpected Worker Separations. Working Paper.

Blatter, M., S. Muehlemann, and S. Schenker (2012). The Costs of Hiring Skilled Workers. *European Economic Review 56*(1), 20–35.

Bloesch, J., B. Larsen, and B. Taska (2022). Which Workers Earn More at Productive Firms? Position Specific Skills and Individual Worker Hold-up Power. Working Paper.

Bloesch, J., S. J. Lee, and J. Weber (2024). Firm Wage Setting and On-the-Job Search Limit Wage-Price Spirals. *Available at SSRN 4734451*.

Bloesch, J. and J. Weber (2023). Congestion in Onboarding Workers and Sticky R&D. *FRB of New York Staff Report* (1075).

Bloom, N., F. Guvenen, B. S. Smith, J. Song, and T. von Wachter (2018). The Disappearing Large-Firm Wage Premium. In *AEA Papers and Proceedings*, Volume 108, pp. 317–22.

Borusyak, K., P. Hull, and X. Jaravel (2022). Quasi-Experimental Shift-Share Research Designs. *The Review of Economic Studies 89*(1), 181–213.

Burdett, K. and D. T. Mortensen (1998). Wage Differentials, Employer Size, and Unemployment. *International Economic Review 39*(2), 257–273.

Burdett, K. and T. Vishwanath (1988). Balanced Matching and Labor Market Equilibrium. *Journal of Political Economy 96*(5), 1048–1065.

Burks, S. V., B. Cowgill, M. Hoffman, and M. Housman (2015). The Value of Hiring through Employee Referrals. *The Quarterly Journal of Economics 130*(2), 805–839.

Caldwell, S. and N. Harmon (2019). Outside Options, Bargaining, and Wages: Evidence from Coworker Networks. Working paper.

Card, D. (2022). Who Set Your Wage? *American Economic Review 112*(4), 1075–1090.

Card, D., A. R. Cardoso, J. Heining, and P. Kline (2018). Firms and Labor Market Inequality: Evidence and Some Theory. *Journal of Labor Economics 36*(S1), S13–S70.

Carrillo-Tudela, C., H. Gartner, and L. Kaas (2023). Recruitment Policies, Job-Filling Rates, and Matching Efficiency. *Journal of the European Economic Association 21*(6), 2413–2459.

Carvalho, M., J. Galindo da Fonseca, and R. Santarrosa (2024). How Are Wages Determined?: A Quasi-Experimental Test of Wage Determination Theories. Working paper.

Chan, M., S. Salgado, and M. Xu (2023). Heterogeneous Passthrough from TFP to Wages. *Available at SSRN 3538503*.

Coles, M. G. (2001). Equilibrium Wage Dispersion, Firm Size, and Growth. *Review of Economic Dynamics 4*(1), 159–187.

Coles, M. G. and D. T. Mortensen (2016). Equilibrium Labor Turnover, Firm Growth, and Unemployment. *Econometrica 84*(1), 347–363.

Datta, N. (2023). The Measure of Monopsony: The Labour Supply Elasticity to the Firm and its Constituents. Working Paper.

Davis, S. J., R. J. Faberman, and J. C. Haltiwanger (2013). The Establishment-Level Behavior of Vacancies and Hiring. *The Quarterly Journal of Economics 128*(2), 581–622.

Dhyne, E., A. K. Kikkawa, T. Komatsu, M. Mogstad, and F. Tintelnot (2022). Foreign Demand Shocks to Production Networks: Firm Responses and Worker Impacts. No. w 30447. National Bureau of Economic Research.

Dube, A., E. Freeman, and M. Reich (2010). Employee Replacement Costs. Working paper.

Dustmann, C., A. Glitz, U. Schönberg, and H. Brücker (2016). Referral-Based Job Search Networks. *The Review of Economic Studies 83*(2), 514–546.

Elsby, M. W., B. Hobijn, and A. Şahin (2013). The Decline of the US Labor Share. *Brookings Papers on Economic Activity 2013*(2), 1–63.

Engbom, N., C. Moser, and J. Sauermann (2023). Firm Pay Dynamics. *Journal of Econometrics 233*(2), 396–423.

Garin, A. and F. Silvério (2024). How Responsive Are Wages to Firm-Specific Changes in Labour Demand? Evidence from Idiosyncratic Export Demand Shocks. *The Review of Economic Studies 91*(3), 1671–1710.

Gavazza, A., S. Mongey, and G. L. Violante (2018). Aggregate Recruiting Intensity. *American Economic Review 108*(8), 2088–2127.

Gouin-Bonenfant, É. (2022). Productivity Dispersion, Between-Firm Competition, and the Labor Share. *Econometrica 90*(6), 2755–2793.

Hall, R. E. and A. I. Mueller (2018). Wage Dispersion and Search Behavior: The Importance of Nonwage Job Values. *Journal of Political Economy 126*(4), 1594–1637.

Heise, S. and T. Porzio (2022). Labor Misallocation Across Firms and Regions. Working Paper.

Hirsch, B., E. J. Jahn, A. Manning, and M. Oberfichtner (2022). The Wage Elasticity of Recruitment. Working Paper.

Hummels, D., R. Jørgensen, J. Munch, and C. Xiang (2014). The Wage Effects of Offshoring: Evidence from Danish Matched Worker-Firm Data. *American Economic Review 104*(6), 1597–1629.

Jäger, S. and J. Heining (2022). How Substitutable are Workers? Evidence from Worker Deaths. NBER Working Paper No. w30629.

Jahn, E. and M. Neugart (2020). Do Neighbors Help Finding a Job? Social Networks and Labor Market Outcomes After Plant Closures. *Labour Economics 65*, 101825.

Jordà, Ò. (2005, March). Estimation and Inference of Impulse Responses by Local Projections. *American Economic Review 95*(1), 161–182.

Jordà, Ò., K. Knoll, D. Kuvshinov, M. Schularick, and A. M. Taylor (2019). The Rate of Return on Everything, 1870–2015. *The Quarterly Journal of Economics 134*(3), 1225–1298.

Kaas, L. and P. Kircher (2015). Efficient Firm Dynamics in a Frictional Labor Market. *American Economic Review 105*(10), 3030–3060.

Kline, P., N. Petcova, H. Williams, and O. Zidar (2019). Who Profits from Patents? Rent-Sharing at Innovative Firms. *The Quarterly Journal of Economics 1343*, 1404.

Koh, D., R. Santaeulàlia-Llopis, and Y. Zheng (2020). Labor Share Decline and Intellectual Property Products Capital. *Econometrica 88*(6), 2609–2628.

Kuhn, P. (2004). Is Monopsony the Right Way to Model Labor Markets? A Review of Alan Manning's Monopsony in Motion. *International Journal of the Economics of Business 11*(3), 369–378.

Lamadon, T., M. Mogstad, and B. Setzler (2022). Imperfect Competition, Compensating Differentials, and Rent Sharing in the US Labor Market. *American Economic Review 112*(1), 169–212.

Langella, M. and A. Manning (2021). Marshall lecture 2020: The Measure of Monopsony. *Journal of the European Economic Association 19*(6), 2929–2957.

Manning, A. (2003). *Monopsony in Motion*. Princeton University Press.

Manning, A. (2006). A Generalised Model of Monopsony. *The Economic Journal 116*(508), 84–100.

Manning, A. (2011). Imperfect Competition in the Labor Market. In *Handbook of Labor Economics*, Volume 4, pp. 973–1041. Elsevier.

Manning, A. (2021). Monopsony in Labor Markets: A Review. *ILR Review 74*(1), 3–26.

Manning, A. (2025). The Immobile Incumbent Problem in a Model of Short-Term Wage-Posting. *German Economic Review* (0).

Matsudaira (2014). Monopsony in the Low-Wage Labor Market? Evidence from Minimum Nurse Staffing Regulations. *The Review of Economics and Statistics 96*(1), 92–102.

Michaillat, P. and E. Saez (2024). $u^* = \sqrt{uv}$: The Full-Employment Rate of Unemployment in the United States. Brookings Papers on Economic Activity Conference Draft, Fall.

Mongey, S. and G. L. Violante (2025). Macro Recruiting Intensity from Micro Data. *American Economic Journal: Macroeconomics, Forthcoming*.

Muehlemann, S. and H. Pfeifer (2016). The Structure of Hiring Costs in Germany: Evidence from Firm-Level Data. *Industrial Relations: A Journal of Economy and Society 55*(2), 193–218.

Rognlie, M. (2015). Deciphering the Fall and Rise in the Net Capital Share: Accumulation or Scarcity? *Brookings Papers on Economic Activity 2015*(1), 1–69.

Rognlie, M. (2018). Comment on "Accounting for Factorless Income". In *NBER Macroeconomics Annual 2018, volume 33*, pp. 235–248. University of Chicago Press.

Schmieder, J. F. (2023). Establishment Age and Wages. *Journal of Econometrics 233*(2), 424–442.

Schubert, G., A. Stansbury, and B. Taska (2023). Employer Concentration and Outside Options. Working Paper.

Seegmiller, B. (2021). Valuing Labor Market Power: The Role of Productivity Advantages. *Available at SSRN 4412667*.

Sokolova, A. and T. Sorensen (2021). Monopsony in Labor Markets: A Meta-Analysis. *ILR Review 74*(1), 27–55.

Sorkin, I. (2018). Ranking Firms using Revealed Preference. *The Quarterly Journal of Economics 133*(3), 1331–1393.

Stevens, M. (2004). Wage-tenure Contracts in a Frictional LabourMmarket: Firms' Strategies for Recruitment and Retention. *The Review of Economic Studies 71*(2), 535–551.

Tanaka, S., L. Warren, and D. Wiczer (2023). Earnings Growth, Job Flows and Churn. *Journal of Monetary Economics 135*, 86–98.

A Appendix

A.1 Estimation Details

We begin by explaining our estimation strategy using minimum distance. Define Z_t to be a vector of a finite sample of $z_{j,t}$ shocks in the model.

In response to a shock, the firm's path for the new hire wages, hiring rate, and employment are deterministic and satisfy (13). We also know that in the linearized model, each variable is a linear function of shocks. For example, $\tilde{w}_{j,t} = \beta_0^w z_{j,t}$. Thus we can substitute firm-level endogenous variables with the firm j-specific shock $z_{j,t}$ times a variable-specific slope:

$$\beta_0^w z_{j,t} = \frac{1}{2\Omega}\left((\chi\beta_0^h z_{j,t} + \sigma\beta_{t-1}^N z_{j,t}) \ + \ S\sum_{\kappa=0}^{\infty}(1-S)^\kappa \, \chi\beta_{\kappa+1}^h z_{j,t} + \sigma\beta_\kappa^N z_{j,t} - (1+\chi)\frac{\gamma}{2}\beta_{\kappa+1}^w z_{j,t} \right),$$

If this equation holds with equality for any individual firm j, summing over j on both sides of the equation also holds with equality, so we can cancel the sum of shocks, $\sum_j z_{j,t}$:

$$\sum_j \beta_0^w z_{j,t} = \sum_j \frac{1}{2\Omega}\left((\chi\beta_0^h z_{j,t} + \sigma\beta_{t-1}^N z_{j,t}) \ + \ S\sum_{\kappa=0}^{\infty}(1-S)^\kappa \, \chi\beta_{\kappa+1}^h z_{j,t} + \sigma\beta_\kappa^N z_{j,t} - (1+\chi)\frac{\gamma}{2}\beta_{\kappa+1}^w z_{j,t} \right)$$

$$\beta_0^w = \frac{1}{2\Omega}\left((\chi\beta_0^h + \sigma\beta_{t-1}^N) \ + \ S\sum_{\kappa=0}^{\infty}(1-S)^\kappa \, \chi\beta_{\kappa+1}^h + \sigma\beta_\kappa^N - (1+\chi)\frac{\gamma}{2}\beta_{\kappa+1}^w \right). \quad (25)$$

Suppose we did not know χ and σ, but we had simulated data from the linearized model. Define $\tilde{W}_t$ as the vector of values $\tilde{w}_{jt}$ for the shocked firms, and $\tilde{W}_{t+1}$ as the vector of values $\tilde{w}_{j,t+1}$ for the shocked firms, $\tilde{\mathcal{H}}_t$ as the vector of log deviations of hiring rates of firms shocked in period t, and $\tilde{\mathcal{N}}_t$ as the vector of log deviations of employment levels of firms shocked in period t, and so on. Let $\tilde{\mathcal{Y}}_t \equiv [\tilde{W}_t \quad \tilde{W}_{t+1}... \quad \tilde{\mathcal{H}}_t \quad \tilde{\mathcal{H}}_{t+1}... \quad \tilde{\mathcal{N}}_t \quad \tilde{\mathcal{N}}_{t+1}...]$ be a matrix of log deviations of steady state of firms shocked in period t. The slope coefficients $\pi \equiv [\beta_0^w \quad \beta_1^w... \quad \beta_0^h \quad \beta_1^h... \quad \beta_0^N \quad \beta_1^N... \quad]$ could be recovered from regressing firm outcomes $\tilde{\mathcal{Y}}_t$ on Z_t:

$$\pi = (Z_t'Z_t)^{-1}(Z_t'\tilde{\mathcal{Y}}_t).$$

Next we can use these coefficients to uncover the true values of χ and σ. Equation (13) can be rolled forward so that we have two conditions that must hold:

$$\beta_0^w + (1+\chi)\frac{\gamma}{4\Omega}S\sum_{\kappa=0}^{\infty}(1-S)^\kappa\beta_{\kappa+1}^w = \frac{\chi}{2\Omega}\left(\beta_0^h + S\sum_{\kappa=0}^{\infty}(1-S)^\kappa\beta_{\kappa+1}^h\right) + \frac{\sigma}{2\Omega}\left(\beta_{-1}^N + S\sum_{\kappa=0}^{\infty}(1-S)^\kappa\beta_\kappa^N\right)$$

$$\beta_1^w + (1+\chi)\frac{\gamma}{4\Omega}S\sum_{\kappa=1}^{\infty}(1-S)^\kappa\beta_{\kappa+1}^w = \frac{\chi}{2\Omega}\left(\beta_1^h + S\sum_{\kappa=1}^{\infty}(1-S)^\kappa\beta_{\kappa+1}^h\right) + \frac{\sigma}{2\Omega}\left(\beta_0^N + S\sum_{\kappa=1}^{\infty}(1-S)^\kappa\beta_\kappa^N\right).$$

We have two equations and two unknowns, and χ and σ can be solved for exactly.

Next, we estimate (14) and (15) in a stacked regression and recover the vector of regression coefficients $\hat{\pi} \equiv [\hat{\beta}_0^w \;\; \hat{\beta}_1^w \;\; \hat{\beta}_2^w \;\; \hat{\beta}_0^h/\bar{h} \;\; \hat{\beta}_1^h/\bar{h} \;\; \hat{\beta}_2^h/\bar{h} \;\; \hat{\beta}_0^N \;\; \hat{\beta}_1^N \;\; \hat{\beta}_2^N]$ from Table 4 of the main text and the following variance-covariance matrix of regression coefficients in Table 10, truncating the regression to have three years of horizon.

Table 10: Variance Covariance Matrix $\hat{\Sigma}$

$\hat{\beta}_0^h$	$\hat{\beta}_1^h$	$\hat{\beta}_2^h$	$\hat{\beta}_0^N$	$\hat{\beta}_1^N$	$\hat{\beta}_2^N$	$\hat{\beta}_0^w$	$\hat{\beta}_1^w$	$\hat{\beta}_2^w$
0.0026	0.0015	0.0013	0.0006	0.0010	0.0010	0.0000	0.0000	0.0000
0.0015	0.0028	0.0015	-0.0003	0.0007	0.0010	0.0000	0.0000	0.0000
0.0013	0.0015	0.0025	-0.0003	-0.0001	0.0006	-0.0001	0.0000	0.0000
0.0006	-0.0003	-0.0003	0.0013	0.0013	0.0011	0.0000	0.0000	0.0000
0.0010	0.0007	-0.0001	0.0013	0.0024	0.0023	0.0001	0.0000	0.0000
0.0010	0.0010	0.0006	0.0011	0.0023	0.0032	0.0000	0.0001	0.0000
0.0000	0.0000	-0.0001	0.0000	0.0001	0.0000	0.0016	-0.0001	-0.0001
0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	-0.0001	0.0016	-0.0002
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0001	-0.0002	0.0015

This table reports the variance-covariance matrix of regression coefficients for the joint regressions of (14) and (15) where the dependent variables are switcher wage growth, firm hiring rate, and log firm employment, in years 0, 1, 2 in response to an export demand shock.

Next, we construct f from equation (17) in the main text. To map our annual regression estimates into a monthly model, we assume that hires occur evenly across months within the year, and plug in annual estimates for each year's corresponding 12 months. The formula for this can be seen in equation (26). We then plug in our point estimates $\hat{\pi}$ into f, and search over values of σ and χ to minimize

$$\min_{\sigma,\chi} f(\hat{\pi}, \sigma, \chi, \Theta)' f(\hat{\pi}, \sigma, \chi, \Theta).$$

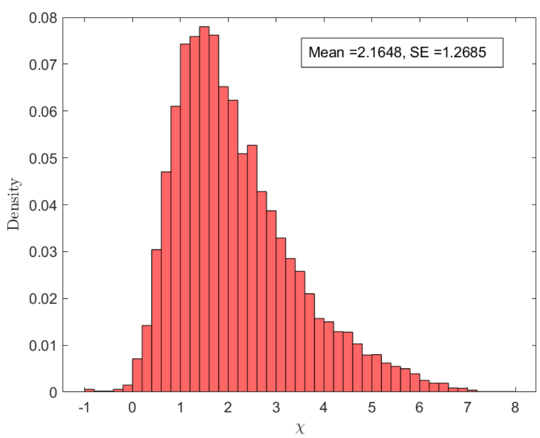
with $\Theta = \{S, \gamma\}$, and where f is written out in equation (26) with $\Omega = \left(1 + \frac{\gamma}{2} + \frac{\gamma(1+\chi)}{4}\right)$. When minimizing (17), we additionally include a small quadratic penalty term: $(12 \times .0025 \times (\chi - 0))^2 + (12 \times .0025 \times (\sigma - 0))^2$, which helps rule out economically implausible estimates. We set $\gamma = 4$ and choose λ_{EE} such that $S = 0.05$. We use the Matlab function lsqnonlin. Our estimate for the inverse labor supply elasticity is then $\hat{\varepsilon}_{w,N} = \frac{\hat{\sigma}}{1+\gamma(1+\hat{\chi})}$.

$$f(\hat{\pi},\sigma,\chi,\Theta)=\underbrace{\begin{bmatrix}\frac{-(1+\chi)\gamma S\sum_{\kappa=0}^{10}(1-S)^{\kappa}}{4\Omega}-1 & \frac{-(1+\chi)S\sum_{\kappa=0}^{9}(1-S)^{\kappa}}{4\Omega}-1 & \dots & 0\\ \frac{-(1+\chi)\gamma S(1-S)^{11}}{4\Omega} & \frac{-(1+\chi)\gamma S\sum_{\kappa=10}^{11}(1-S)^{\kappa}}{4\Omega} & \dots & -1\\ 0 & 0 & \dots & \frac{-(1+\chi)\gamma S\sum_{\kappa=0}^{11}(1-S)^{\kappa}}{4\Omega}\\ \frac{\chi\left(1+S\sum_{\kappa=0}^{10}(1-S)^{\kappa}\right)}{2\Omega} & \frac{\chi\left(1+S\sum_{\kappa=0}^{9}(1-S)^{\kappa}\right)}{2\Omega} & \dots & 0\\ \frac{\chi S(1-S)^{11}}{2\Omega} & \frac{\chi S\sum_{\kappa=10}^{11}(1-S)^{\kappa}}{2\Omega} & \dots & \frac{\chi}{2\Omega}\\ 0 & 0 & \dots & \frac{\chi S\sum_{\kappa=0}^{11}(1-S)^{\kappa}}{2\Omega}\\ \frac{\sigma\left(S\sum_{\kappa=0}^{11}(1-S)^{\kappa}\right)}{2\Omega} & \frac{\sigma\left(1+S\sum_{\kappa=0}^{10}(1-S)^{\kappa}\right)}{2\Omega} & \dots & 0\\ 0 & \frac{\sigma S\sum_{\kappa=11}^{11}(1-S)^{\kappa}}{2\Omega} & \dots & \frac{\sigma}{2\Omega}\\ 0 & 0 & \dots & \frac{\sigma S\sum_{\kappa=0}^{11}(1-S)^{\kappa}}{2\Omega}\end{bmatrix}'}_{A_{24\times 9}}\times\underbrace{\begin{bmatrix}\hat{\beta}_0^w\\ \hat{\beta}_1^w\\ \hat{\beta}_2^w\\ \hat{\beta}_0^h/\bar{h}\\ \hat{\beta}_1^h/\bar{h}\\ \hat{\beta}_2^h/\bar{h}\\ \hat{\beta}_0^N\\ \hat{\beta}_1^N\\ \hat{\beta}_2^N\end{bmatrix}}_{\hat{\pi}_{9\times 1}} \tag{26}$$

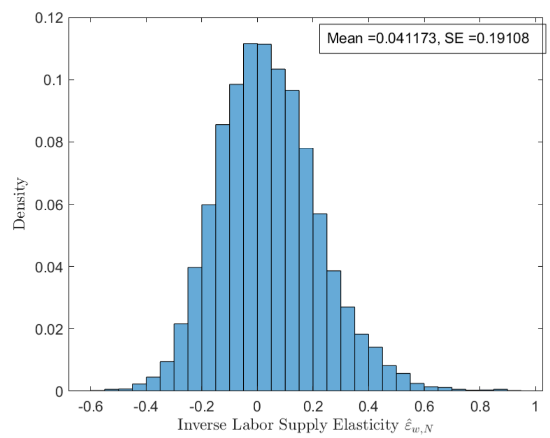
Lastly, we characterize the uncertainty of our estimates by sampling from a joint normal distribution $\sim \mathcal{N}(\hat{\pi},\hat{\Sigma})$. For each draw, we re-estimate σ and χ using the same procedure and compute an inverse labor supply elasticity $\hat{\varepsilon}_{w,N}=\frac{\hat{\sigma}}{1+\gamma(1+\hat{\chi})}$ for each draw. We plot the distribution of $\hat{\chi}$ and $\hat{\varepsilon}_{w,N}$ in Figure 5 and report the 10th and 90th percentiles of $\hat{\chi}$, $\hat{\sigma}$, and $\hat{\varepsilon}_{w,N}$ in Table 5 in the main text.

Figure 5: Distribution Estimates of χ and Inverse Labor Supply Elasticity $\varepsilon_{w,N}$

(a) Recruiting Rate Cost Parameter $\hat{\chi}$



(b) Inverse Labor Supply Elasticity $\hat{\varepsilon}_{w,N}$

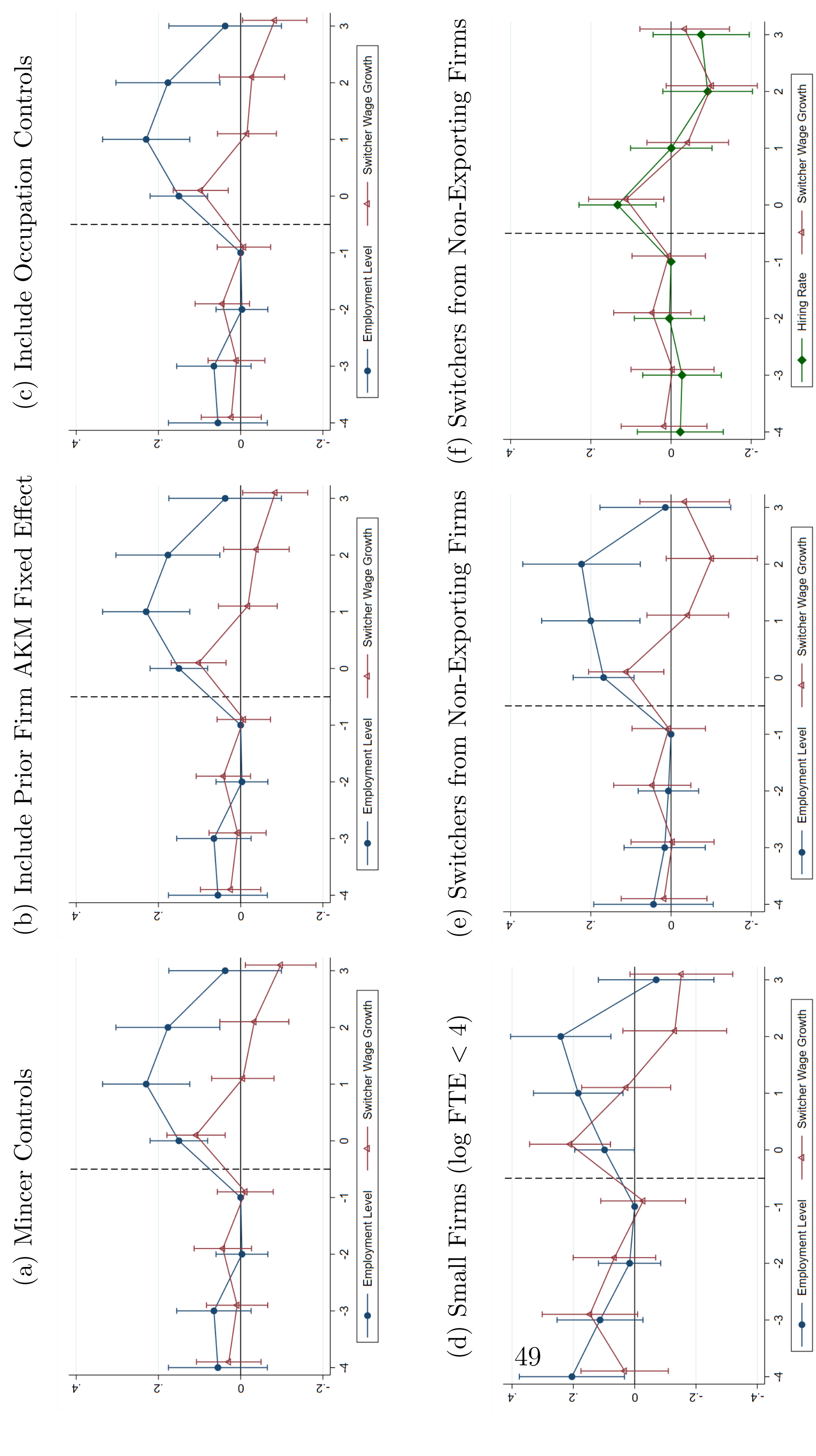


This figure shows the distribution of estimates for the parameter χ and firm specific inverse labor supply elasticity $\varepsilon_{w,N}$. Estimates are obtained by resampling from a multivariate normal distributed $\mathcal{N}(\hat{\pi},\hat{\Sigma})$, and choosing σ and χ to minimize equation (17), where f is defined in equation (26).

B Online Appendix

B.1 Robustness: Measures of Switcher Wage Growth

Figure 6: Robustness: Employment, Hiring, and Switcher Wage Growth Response



This figure reports robustness to Figure 3a. In panel (a), switcher wages are constructed by first residualized switcher wage growth on mincer controls, and then taking the average residualized switcher wage at the firm-year level. In panel (b), switcher wage growth residuals are constructed taking into the AKM fixed effect of the firm that that workers leave, in addition to Mincer controls. In panel (c), switcher wage growth residuals are constructed taking in account Mincer Controls, the prior firm AKM fixed effect, and a variable that estimates the average wage change when a worker switches between two occupations. Panel (d) uses the same measure as Panel (b), but using only firms whose log full time equivalent employment was less than 4 (approximately 55 full-time equivalent workers) in the year prior to the shock. Panels (e) and (f) construct new hire wage growth using only workers who switch from non-exporting firms.

B.2 Worker's Problem

Worker's Value Function The worker gets utility from wages and draws a utility shock when given a choice between two firms. Let ν_k be the share of vacancies in the market posting wage $w = w_k$.

$$
\begin{aligned}
\mathcal{V}_{j,s,t} = \frac{\eta}{\eta-1}\left(w_{j,s,t}^{\frac{\eta-1}{\eta}} - 1\right) + \beta_w s_0 \mathcal{V}_U + \\
\beta_w(1-s_0)\mathbb{E}_t\Bigg[\lambda_{EE} f(\theta)\int_k \nu_k \gamma^{-1}\log\Bigg(\exp\left(\gamma\mathcal{V}_{j,s,t+1}\right) + \exp\left(\gamma\mathcal{V}_{k,t+1}\right)\Bigg)dk + \\
(1-\lambda_{EE} f(\theta))\max\{\mathcal{V}_{j,s,t+1}, \mathcal{V}_U\}\Bigg],
\end{aligned}
\tag{27}
$$

How Worker's Value Function Changes with a Firm's Wage When deriving how a change in the firm's wage policy affects worker values, we will assume that workers believe that the wage change is permanent. We will consider the case of no wage dispersion and $\mathcal{V}_U < \mathcal{V}_j$:

$$
\frac{\partial \mathcal{V}_j}{\partial w_j} = w_j^{-\frac{1}{\eta}} + \beta_w(1-s_0)\left(f(\theta)\lambda_{EE}\int \upsilon_k\left(\frac{\exp\left(\gamma\mathcal{V}_j\right)}{\exp\left(\gamma\mathcal{V}_j\right)+\exp\left(\gamma\mathcal{V}_k\right)}\right)dk + \Big(1-f(\theta)\lambda_{EE}\Big)\right)\frac{\partial \mathcal{V}_j}{\partial w_j}.
$$

We can substitute in the expression for the separation rate from equation (3) (under the assumption that workers are not quitting into unemployment or exogenously separating). Under symmetry, values are equal across firms $\mathcal{V}_j = \mathcal{V}_k$, firm employment and firm vacancy shares are equal $\phi_k = \upsilon_k$, and if there are no exogenous separations, then $S = \lambda_{EE} f(\theta)/2$.

$$
\begin{aligned}
\frac{\partial \mathcal{V}_j}{\partial w_j} &= w_j^{-\frac{1}{\eta}} + \beta_w\left(S(w_j, w_{-j}, \theta) + \Big(1-f(\theta)\lambda_{EE}\Big)\right)\frac{\partial \mathcal{V}_j}{\partial w_j} \\
&= w_j^{-\frac{1}{\eta}} + \beta_w\left(S(w_j, w_{-j}, \theta) + \Big(1-2S(w_j, w_{-j}, \theta)\Big)\right)\frac{\partial \mathcal{V}_j}{\partial w_j} \\
&= w_j^{\frac{-1}{\eta}} + \beta_w\Big(1-S(w_j, w_{-j}, \theta)\Big)\frac{\partial \mathcal{V}_j}{\partial w_j},
\end{aligned}
$$

resulting in

$$
\frac{\partial \mathcal{V}_j}{\partial w_j} = \frac{w_j^{-\frac{1}{\eta}}}{(1-\beta_w(1-S))}. \tag{28}
$$

B.3 Firms' Dynamic Problem

This appendix section derives the decomposition of marginal product in Section 2.3, quantifies the effect of non-zero discounting, and derives the inverse labor supply elasticity in Section 2.3.

If the firm is operating within a stationary environment, (i.e., $S(w)$ and $R(w)$ are not changing), the firm maximizes the present discounted value of profits. Ignoring a firm subscript, the firm's problem is:

$$\max_{\left\{V_t,\, K_t,\, N_t,\, \{N_{s,t}\}_{s\leq t},\, \{w_{s,t}\}_{s\leq t}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_f^t \left(A_t \left(K_t^{\alpha} N_t^{1-\alpha}\right)^{\frac{\epsilon-1}{\epsilon}} - \sum_{s=-\infty}^{t} w_{s,t} N_{s,t} - r^K K_t - c\left(\frac{V_t}{N_{t-1}}\right)^{\chi} N_{t-1}^{\sigma} V_t \right) \tag{29}$$

subject to

$$\begin{aligned}
\text{Recruiting:} \quad & N_{t,t} = R(w_{t,t})V_t \quad \text{for all } t \\
\text{Retention:} \quad & N_{s,t} = (1 - S(w_{s,t}))N_{s,t-1} \quad \text{for all } t \text{ and } s < t \\
\text{Employment is sum of cohorts:} \quad & N_t = \sum_{s=-\infty}^{t} N_{s,t} \quad \text{for all } t \\
\text{Wage at least posted wage:} \quad & w_{s,t} \geq w_{s,s} \quad \text{for all } t \text{ and } s < t
\end{aligned}$$

where the total number of workers in the firm N_t is the sum of all workers from each past cohort still remaining in the firm: $N_t = \sum_{s=-\infty}^{t} N_{s,t}$ which we substitute into the equation and obtain the Lagrangian and check for the fourth restriction in Appendix B.5.3:

$$\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta_f^t \Bigg[& A_t \left(\sum_{s=-\infty}^{t} N_{s,t}\right)^{\alpha} - \sum_{s=-\infty}^{t} w_{s,t} N_{s,t} - c\left(\frac{V_t}{\sum_{s=-\infty}^{t-1} N_{s,t-1}}\right)^{\chi} \left(\sum_{s=-\infty}^{t-1} N_{s,t-1}\right)^{\sigma} V_t \\
& + \vartheta_t \left(R(w_{t,t})V_t - N_{t,t}\right) \\
& + \sum_{s<t} \lambda_{s,t} \left((1 - S(w_{s,t}))N_{s,t-1} - N_{s,t}\right) \Bigg]
\end{aligned}$$

The first order conditions are:

$$\begin{aligned}
\mathcal{L}_{N_{s,t}}: \quad & \alpha A_t N_t^{\alpha-1} - w_{s,t} - \lambda_{s,t} + \beta_f c(\chi - \sigma)\left(\frac{V_{t+1}}{N_t}\right)^{\chi} N_t^{\sigma-1} V_{t+1} + \beta_f \lambda_{s,t+1}(1 - S(w_{s,t+1})) = 0 \\
\mathcal{L}_{w_{s,t}}: \quad & -N_t - \lambda_{s,t} S'(w_{s,t}) N_{t-1} + \vartheta_t R'(w_{s,t}) V_t = 0 \\
\mathcal{L}_{V_t}: \quad & -c(1+\chi)\left(\frac{V_t}{N_{t-1}}\right)^{\chi} N_{t-1}^{\sigma} + \vartheta_t R(w_{t,t}) = 0.
\end{aligned}$$

Rearranging the first order condition on V_t yields, imposing steady state, and using that $\frac{V}{N} = \frac{S(w)}{R(w)}$ whereby also $\vartheta{=}\lambda$:

$$\vartheta = \lambda = \frac{c(1+\chi)\left(\frac{S(w)}{R(w)}\right)^{\chi}}{R(w)} N^{\sigma}.$$

Using the expressions for μ and the first order condition on wages yields

$$w = c(1+\chi)\left(\frac{S(w)}{R(w)}\right)^{1+\chi}(\varepsilon_{R,w} - \varepsilon_{S,w})N^{\sigma} = c(1+\chi)\left(\frac{S(w)}{R(w)}\right)^{1+\chi}\mathcal{E}N^{\sigma}. \tag{30}$$

The steady state value of employment N can be solved for using the previous two expressions and the first order condition on employment. The choice of firm size N will depend on the discount factor, as growing requires upfront costs, so firms that discount more steeply will choose to be smaller. Optimal employment is:

$$\alpha A N^{\alpha-1} = \frac{w}{\mathcal{E}(1+\chi)}\left(1 + \mathcal{E}(1+\chi) + \sigma + (1-\beta_f)\left(\chi - \sigma + (1+\chi)\frac{1-S(w)}{S(w)}\right)\right). \tag{31}$$

In steady state, the level of vacancies is given by

$$V = \frac{S(w)}{R(w)}N. \tag{32}$$

Collectively, equations (30), (31), and (32) characterize the firm's optimal choice of wages, employment, and vacancies in steady state.

With these expressions, we can solve for what shares of marginal product go to wages, turnover costs, and profits in steady state. The marginal revenue product of labor is $MRPL = \alpha A N^{\alpha-1}$. The wage markdown is then

$$\frac{w}{MRPL} = \frac{\mathcal{E}(1+\chi)}{1 + \mathcal{E}(1+\chi) + \sigma + (1-\beta_f)\left(\chi - \sigma + (1+\chi)\frac{1-S(w)}{S(w)}\right)}.$$

Recruiting costs per worker are given by $c\big(V/N\big)^{\chi+1}N^{\sigma}$ in steady state, so as a share of marginal product we get

$$\frac{\text{Recruiting costs per worker}}{MRPL} = \frac{1}{1 + \mathcal{E}(1+\chi) + \sigma + (1-\beta_f)\left(\chi - \sigma + (1+\chi)\frac{1-S(w)}{S(w)}\right)}.$$

The labor market profit per worker is the gap between marginal product and the sum of wages and per-incumbent recruiting costs.

$$\frac{\text{Labor market profits per worker}}{MRPL} = \frac{\sigma + (1-\beta_f)\left(\chi - \sigma + (1+\chi)\frac{1-S(w)}{S(w)}\right)}{1 + \mathcal{E}(1+\chi) + \sigma + (1-\beta_f)\left(\chi - \sigma + (1+\chi)\frac{1-S(w)}{S(w)}\right)}.$$

How large are the additional terms due to discounting? At a monthly frequency, total monthly separation rates are approximately 0.04, and given an annual discount rate, β_f can be approximated to being equal to 0.996. Setting $\sigma = 0$ and $\chi = 2$, the term in the denominator is

$$(1-\beta_f)\left(\chi - \sigma + (1+\chi)\frac{1-S(w)}{S(w)}\right) = (1-0.996) \times (2 + 3 \times \frac{0.96}{0.04}) \approx 0.296.$$

With $\mathcal{E} = 4$, the profit per worker as a share of marginal product collected by firms due to discounting is then (with $\sigma = 0$):

$$\frac{0.296}{1 + 4(1+2) + 0.296} \approx 0.022.$$

This calculation implies that under standard parameters, the additional profits collected on the margin are 2.2% of marginal product. Economically, standard time preferences are quantitatively unimportant because the discounting from the time preference rate is an order of magnitude smaller than the separation rate. Because matching with a worker requires an upfront investment with a future flow of payoffs, but the life of that flow of payoffs is affected by the separation rate, the relevant discount rate to the firm is the sum of the separation rate and time preference parameter.

Deriving the Inverse Labor Supply Elasticity in General Form To derive the inverse labor supply elasticity, we start with the optimal wage equation:

$$w = c(1+\chi)\left(\frac{S(w)}{R(w)}\right)^{1+\chi}(\varepsilon_{R,w} - \varepsilon_{S,w})\, N^{\sigma}.$$

Taking logs (and) reinstating the (w)) yields

$$\log(w) = \log(c(1+\chi)) + (1+\chi)\log\left(\frac{S(w)}{R(w)}\right) + \log\mathcal{E} + \sigma\log(N).$$

where $\mathcal{E} = \varepsilon_{R,w} - \varepsilon_{S,w}$. Taking the total derivative with respect to $\log(w)$, we get

$$\begin{aligned} 1 &= (1+\chi)(\varepsilon_{S,w} - \varepsilon_{R,w}) + \varepsilon_{\mathcal{E},w} + \sigma\varepsilon_{N,w} \\ &= -(1+\chi)\mathcal{E} + \varepsilon_{\mathcal{E},w} + \sigma\varepsilon_{N,w}. \end{aligned}$$

Rearranging and using $\varepsilon_{w,N} = \varepsilon_{N,w}^{-1}$ yields

$$\varepsilon_{w,N} = \frac{\sigma}{1 + (1+\chi)\mathcal{E} - \varepsilon_{\mathcal{E},w}}.$$

B.4 Recruiting Costs as Reallocating Labor

In this section, we show that counting recruiting costs as taking away from labor inputs generates identical formulas as treating recruiting costs as expenditure. Suppose that recruiting takes away from labor inputs, so a firm's labor input is

$$N_t - cV_t^{1+\chi} N_{t-1}^{\sigma-\chi}.$$

For simplicity, assume $\sigma = 0$. If firms do not discount ($\beta_f = 1$) and revenue is $PY = \left(K_t^\alpha \left(N_t - cV_t^{1+\chi} N_{t-1}^{-\chi}\right)^{1-\alpha}\right)^{\frac{\epsilon-1}{\epsilon}}$, then firm maximizes the flow rate of profits:

$$\left(K_t^\alpha \left(N_t - cV_t^{1+\chi} N_{t-1}^{-\chi}\right)^{1-\alpha}\right)^{\frac{\epsilon-1}{\epsilon}} - w_t N_t - r^K K_t.$$

Given that $V_t = \frac{S(w_t)}{R(w_t)} N_t$ in steady state, and dropping time subscripts, we have that the firm solves

$$\max_{K,N,w} \left(K^\alpha \left(N - c\left(\frac{S(w)}{R(w)}\right)^{1+\chi} N\right)^{1-\alpha}\right)^{\frac{\epsilon-1}{\epsilon}} - wN - r^K K$$

$$K^{\alpha\frac{\epsilon-1}{\epsilon}} N^{(1-\alpha)\frac{\epsilon-1}{\epsilon}} \left(1 - c\left(\frac{S(w)}{R(w)}\right)^{1+\chi}\right)^{(1-\alpha)\frac{\epsilon-1}{\epsilon}} - wN - r^K K.$$

Taking the first order condition with respect to N, we have

$$FOC_N : (1-\alpha)\frac{\epsilon-1}{\epsilon} K^{\alpha\frac{\epsilon-1}{\epsilon}} N^{(1-\alpha)\frac{\epsilon-1}{\epsilon}-1} \left(1 - c\left(\frac{S(w)}{R(w)}\right)^{1+\chi}\right)^{(1-\alpha)\frac{\epsilon-1}{\epsilon}} = w.$$

Multiplying both sides by N, we have

$$(1-\alpha)\frac{\epsilon-1}{\epsilon} K^{\alpha\frac{\epsilon-1}{\epsilon}} N^{(1-\alpha)\frac{\epsilon-1}{\epsilon}} \left(1 - c\left(\frac{S(w)}{R(w)}\right)^{1+\chi}\right)^{(1-\alpha)\frac{\epsilon-1}{\epsilon}} = wN \tag{33}$$

Rearranging the first order condition on N once again, we come away with the simple formula

$$(1-\alpha)\frac{\epsilon-1}{\epsilon} PY = wN,$$

and consequently the labor share is the same as in the main text:

$$\frac{wN}{PY} = (1-\alpha)\frac{\epsilon-1}{\epsilon}.$$

B.5 Proof of Proposition 1

This appendix section proves Proposition 1 in three steps. First, in Section B.5.1, assuming a stationary equilibrium exists, we show that the sum of recruiting and separation elasticities is equal to γ at any wage level. In Section B.5.2, we characterize firms' optimal wages and employment levels as functions of parameters and aggregate variables. Then in Section B.5.3, we prove that firms do not find it optimal to backload wages. Lastly, in Section B.5.4, we show that the equilibrium is unique.

B.5.1 Constant Sum of Recruiting and Separation Elasticities

This section proves the claim from Section 2.7 that if there are no exogenous separations $s_0 = 0$, workers completely discount $\beta_w = 0$, and workers have log indirect utility from wages $\eta \to 1$, and if a stationary equilibrium exists where firms are in steady state, then the sum of recruiting and separation elasticities $\varepsilon_{R,w} - \varepsilon_{S,w} = \gamma$ for any choice of w in steady state.

Suppose that a steady state equilibrium exists, characterized by tightness θ, the aggregate wage index $\tilde{w}$, the distribution of posted wages $\Upsilon(w)$, the distribution of employed wages $\Phi(w)$, and wage, employment, and vacancy policies w_j^*, N_j^*, and V_j^*. Suppose there are K wage levels. Given $w_1, ...w_K$ and $V_1, ..., V_K$, we want to solve for steady state employment shares $\phi_1, ..., \phi_K$. We also want to show that $\varepsilon_{R,w} - \varepsilon_{S,w} = \gamma$ in this steady state. Let us assume that:

$$\frac{\phi_i}{\phi_k} = \frac{\upsilon_i}{\upsilon_k}\Big(\frac{w_i}{w_k}\Big)^\gamma$$

for any two wage levels i and k. Then we need to (1) show that the employment share in any wage level is in a steady state, (2) solve for the level of the ϕ_k's, and (3) show that $\varepsilon_{R,w} - \varepsilon_{S,w} = \gamma$ for any firm j's wage policy w_j.

First let's show that inflows are equal to outflows for any sector i. Inflows to sector i are:

$$\begin{aligned}
&V_i g(\theta) \sum_{k\neq i} \phi_k \frac{w_i^\gamma}{w_i^\gamma + w_k^\gamma} \\
&= V_i \frac{g(\theta)}{f(\theta)} f(\theta) \sum_{k\neq i} \phi_i \frac{\upsilon_k}{\upsilon_i}\left(\frac{w_k}{w_i}\right)^\gamma \frac{w_i^\gamma}{w_i^\gamma + w_k^\gamma} \quad \left(\text{since } \frac{\phi_k}{\phi_i} = \frac{\upsilon_k}{\upsilon_i}\Big(\frac{w_k}{w_i}\Big)^\gamma\right) \\
&= V_i \frac{1}{\theta} f(\theta) \phi_i \frac{1}{\upsilon_i} \sum_{k\neq i} \upsilon_k \frac{w_k^\gamma}{w_i^\gamma + w_k^\gamma} \\
&= V_i \frac{S}{V} f(\theta) \phi_i \frac{V}{V_i} \sum_{k\neq i} \upsilon_k \frac{w_k^\gamma}{w_i^\gamma + w_k^\gamma} \\
&= SN f(\theta) \phi_i \sum_{k\neq i} \upsilon_k \frac{w_k^\gamma}{w_i^\gamma + w_k^\gamma}
\end{aligned}$$

$$=N_i \lambda_{EE} f(\theta) \sum_{k \neq i} \upsilon_k \frac{w_k^\gamma}{w_i^\gamma + w_k^\gamma},$$

which is the formula for outflows from sector i (using $f(\theta)/g(\theta) = \theta$, $\theta = V/S$, $S = \lambda_{EE} N$, $\phi_i = N_i/N$). Thus, when $\frac{\phi_i}{\phi_k} = \frac{\upsilon_i}{\upsilon_k} \left(\frac{w_i}{w_k} \right)^\gamma$, this labor market is in steady state.

Next we solve for the values of ϕ_i. First, define some constant C such that

$$\frac{\upsilon_i w_i^\gamma}{\phi_i} = \frac{\upsilon_k w_k^\gamma}{\phi_k} = C \rightarrow \phi_k = \frac{\upsilon_k w_k^\gamma}{C}, \ \ \forall k.$$

We also know that $\sum_{k=1}^K \phi_k = 1$. Thus

$$\sum_{k=1}^K \frac{w_k^\gamma \upsilon_k}{C} = 1 \longrightarrow \sum_{k=1}^K w_k^\gamma \upsilon_k = C.$$

Thus

$$\phi_i = \frac{\upsilon_i w_i^\gamma}{C} = \frac{\upsilon_i w_i^\gamma}{\sum_{k=1}^K \upsilon_k w_k^\gamma}.$$

Lastly, we show that for any firm j, the firm faces a constant $\varepsilon_{R,w} - \varepsilon_{S,w} = \gamma$ at any value of w_j. The firm's ratio of separation rate to recruiting rate is:

$$\begin{aligned}\frac{S(w_j)}{R(w_j)} &= \frac{f(\theta)\lambda_{EE}\Big(\upsilon_1 \frac{w_1^\gamma}{w_1^\gamma + w_j^\gamma} + ... + \upsilon_K \frac{w_K^\gamma}{w_K^\gamma + w_j^\gamma}\Big)}{g(\theta)\Big(\phi_1 \frac{w_j^\gamma}{w_1^\gamma + w_j^\gamma} + ... + \phi_K \frac{w_j^\gamma}{w_K^\gamma + w_j^\gamma}\Big)} \\ &= \lambda_{EE}\theta \frac{\Big(\upsilon_1 \frac{w_1^\gamma}{w_1^\gamma + w_j^\gamma} + ... + \upsilon_K \frac{w_K^\gamma}{w_K^\gamma + w_j^\gamma}\Big)}{\Big(\phi_1 \frac{w_j^\gamma}{w_1^\gamma + w_j^\gamma} + ... + \phi_K \frac{w_j^\gamma}{w_K^\gamma + w_j^\gamma}\Big)}\end{aligned}$$

Using that $\upsilon_i w_i^\gamma = \phi_i \sum_{k=1}^K w_k^\gamma \upsilon_k$, we can rewrite the above equation as:

$$\begin{aligned}\frac{S(w_j)}{R(w_j)} &= \lambda_{EE}\theta \frac{\sum_{k=1}^K \upsilon_k w_k^\gamma}{w_j^\gamma} \frac{\Big(\phi_1 \frac{1}{w_1^\gamma + w_j^\gamma} + ... + \phi_K \frac{1}{w_K^\gamma + w_j^\gamma}\Big)}{\Big(\phi_1 \frac{1}{w_1^\gamma + w_j^\gamma} + ... + \phi_K \frac{1}{w_K^\gamma + w_j^\gamma}\Big)} \\ &= \lambda_{EE}\theta w_j^{-\gamma} \sum_{k=1}^K \upsilon_k w_k^\gamma = \lambda_{EE}\theta \Big(\frac{\tilde{w}}{w_j}\Big)^\gamma,\end{aligned}$$

with $\tilde{w} = (\sum_{k=1}^K \upsilon_k w_k^\gamma)^{\frac{1}{\gamma}}$, where the elasticity of this function with respect to w_j is $-\gamma$.

B.5.2 Characterization of Constant Elasticity of Equilibrium

Next, we characterize firms' optimal employment and wages, focusing on the case of $\eta = 1$, $s_0 = 0$, $\beta_f = 1$, and $\beta_w = 0$. By solving out for capital, we can define auxiliary parameters

$$\tilde{A}_j = A_j^{\frac{1}{1-\alpha_j \frac{\epsilon-1}{\epsilon}}} \left(\frac{\alpha_j \frac{\epsilon-1}{\epsilon}}{r^K} \right)^{\frac{\alpha_j \frac{\epsilon-1}{\epsilon}}{1-\alpha_j \frac{\epsilon-1}{\epsilon}}} \qquad \tilde{\alpha}_j = \frac{(1-\alpha_j)\frac{\epsilon-1}{\epsilon}}{1-\alpha_j \frac{\epsilon-1}{\epsilon}}.$$

Given parameters $\tilde{\alpha}_j$, $\tilde{A}_j$, χ, σ, and c_j, aggregate labor market tightness θ, and aggregate wage index $\tilde{w}$, a firm j's optimal wage is

$$w_j^* = \left(c_j \gamma (1+\chi)(\lambda_{EE}\theta)^{1+\chi} \tilde{w}^{\gamma(1+\chi)} \left(\frac{\tilde{\alpha}_j \tilde{A}_j \gamma(1+\chi)}{1+\gamma(1+\chi)+\sigma} \right)^{\frac{\sigma}{1-\tilde{\alpha}_j}} \right)^{\frac{1-\tilde{\alpha}_j}{(1-\tilde{\alpha}_j)(1+\gamma(1+\chi))+\sigma}}, \tag{34}$$

where $\tilde{w}$ is a vacancy-weighted index of the distribution of wages: $\tilde{w} = \left(\int_{w_k} \nu(w_k) w_k^{\gamma} dw_k \right)^{\frac{1}{\gamma}}$. Taking $\beta_f \to 1$, optimal employment at firm j is

$$N_j^* = \left(c_j \gamma (1+\chi)(\lambda_{EE}\theta)^{1+\chi} \tilde{w}^{\gamma(1+\chi)} \right)^{\frac{-1}{(1-\tilde{\alpha}_j)(1+\gamma(1+\chi))+\sigma}} \left(\tilde{\alpha}_j \tilde{A}_j \frac{\gamma(1+\chi)}{(1+\gamma(1+\chi))+\sigma} \right)^{\frac{1+\gamma(1+\chi)}{(1-\tilde{\alpha}_j)(1+\gamma(1+\chi))+\sigma}} \tag{35}$$

To have an interior solution, we must have that $\sigma > -(1-\tilde{\alpha}_j)(1+\gamma(1+\chi))$, i.e., σ cannot be too negative. The optimal vacancy policy is simply $V_j^* = \bar{V}(\tilde{w}/w_j)^{\gamma} N_j^*$, and labor market tightness is $\theta = (\int_{k \in J} V_k dk)/(\lambda_{EE} N)$.

B.5.3 No Backloading of Wages

We want to show that in our equilibrium with myopic workers and no exogenous separations, no firm has an incentive to backload wages. In an equilibrium with log utility $\eta \to 0$, no exogenous separations $s_0 = 0$, and myopic workers $\beta_w = 0$, we have the result that $\varepsilon_{R,w} - \varepsilon_{S,w} = \gamma$ at any wage level. We now want to show it in general for K wages. That is, wages are $w_1, w_2, ..., w_K$, employment shares, $\{\phi_k\}_{k=1}^K$, vacancy shares, $\{\upsilon_k\}_{k=1}^K$ where $\sum_{k=1}^K \phi_k = 1$ and $\sum_{k=1}^K \upsilon_k = 1$.

In equilibrium, the firm's first order condition on wages satisfies

$$N^* = \lambda(R'(w^*)V^* - S'(w^*)N^*),$$

where stars denote the optimal choice variables. Rearranging and plugging in steady-state condition $\frac{V}{N} = \frac{S}{R}$, this yields

$$w^* = S(w^*)\lambda(\varepsilon_{R,w^*} - \varepsilon_{S,w^*}). \tag{36}$$

Now consider a firm that is setting wages period by period. The first order condition yields

$$\begin{aligned}
-S'(w_t)N_{t-1}\lambda =& N_t \\
-S'(w_t)N_{t-1}\lambda =& (1-S(w_t))N_{t-1} \\
-S'(w_t)\lambda =& (1-S(w_t)).
\end{aligned}$$

Let's define $w^{\#}$ as the optimal wage if the firm were setting wages period by period. Then the optimal wage would satisfy

$$w^{\#} = \lambda(-\varepsilon_{S,w^{\#}})\frac{S(w^{\#})}{1-S(w^{\#})}. \tag{37}$$

We know that the left hand side and right hand side of equation (36) cross only once because when we plug in the value of λ into equation (36), we get

$$w = c(1+\chi)\left(\frac{S(w)}{R(w)}\right)^{1+\chi}(\varepsilon_{R,w}-\varepsilon_{S,w})N^{\sigma}.$$

For any firm size N, since $\varepsilon_{R,w}-\varepsilon_{S,w}=\gamma$ and $S(w)/R(w) \propto w^{-\gamma}$, the left and right hand side cross only once.

Thus, we can show that the optimal discretionary wage $w^{\#}$ is below the commitment wage w^* if the right hand side of equation (37) lies below the right hand side of equation (34) for all w, that is if

$$\lambda S(w)(\varepsilon_{R,w}-\varepsilon_{S,w}) + \lambda\varepsilon_{S,w}\frac{S(w)}{1-S(w)} > 0$$

for all w. With some rearranging, this expression becomes

$$\begin{aligned}
\lambda S(w)\varepsilon_{R,w} - \lambda S(w)\varepsilon_{S,w} + \lambda\varepsilon_{S,w}\frac{S(w)}{1-S(w)} &> 0 \\
S(w)(\varepsilon_{R,w}-\varepsilon_{S,w})(1-S(w)) + \varepsilon_{S,w}S(w) &> 0 \\
(\varepsilon_{R,w}-\varepsilon_{S,w})(1-S(w)) + \varepsilon_{S,w} &> 0 \\
(\varepsilon_{R,w}-\varepsilon_{S,w}) + \varepsilon_{S,w} &> (\varepsilon_{R,w}-\varepsilon_{S,w})S(w)
\end{aligned}$$

With $\varepsilon_{R,w}-\varepsilon_{S,w}=\gamma$, this becomes

$$\varepsilon_{R,w} > \gamma S(w).$$

So now we need to prove this condition. Define the separation function for a firm paying wage w:

$$S(w) = \lambda_{EE}f(\theta)\sum_{k=1}^{K}\frac{\bar{w}_k^{\gamma}}{w^{\gamma}+\bar{w}_k^{\gamma}}\upsilon_k.$$

The recruiting function is

$$R(w) = g(\theta)\sum_{k=1}^{K}\frac{\bar{w}^{\gamma}}{w^{\gamma}+\bar{w}_k^{\gamma}}\phi_k.$$

the elasticity is then: The recruiting elasticity is:

$$\varepsilon_{R,w} = \gamma\cdot\frac{\sum_{k=1}^{K}\frac{\bar{w}_k^{\gamma}w^{\gamma}}{(w^{\gamma}+\bar{w}_k^{\gamma})^2}\phi_k}{\sum_{k=1}^{K}\frac{w^{\gamma}}{w^{\gamma}+\bar{w}_k^{\gamma}}\phi_k}.$$

The condition is then

$$\varepsilon_{R,w} > \gamma S(w).$$

If and only if

$$\gamma\frac{\sum_{k=1}^{K}\frac{\bar{w}_k^{\gamma}w^{\gamma}}{(w^{\gamma}+\bar{w}_k^{\gamma})^2}\phi_k}{\sum_{k=1}^{K}\frac{w^{\gamma}}{w^{\gamma}+\bar{w}_k^{\gamma}}\phi_k} > \gamma\lambda_{EE}f(\theta)\sum_{k=1}^{K}\frac{\bar{w}_k^{\gamma}}{w^{\gamma}+\bar{w}_k^{\gamma}}\upsilon_k,$$

where $\lambda_{EE}f(\theta) < 1$. Let

$$\frac{\phi_i}{\phi_k} = \frac{\upsilon_i}{\upsilon_k}\left(\frac{w_i}{w_k}\right)^{\gamma}$$

for any two wage levels i and k.

First, define some constant C such that

$$\frac{\upsilon_i w_i^{\gamma}}{\phi_i} = \frac{\upsilon_k w_k^{\gamma}}{\phi_k} = C,$$

Using the condition that $\sum_{k=1}^{K}\phi_k = 1$, we find:

$$\sum_{k=1}^{K}\frac{\upsilon_k w_k^{\gamma}}{C} = 1 \Rightarrow C = \sum_{k=1}^{K}\upsilon_k w_k^{\gamma}$$

Therefore we obtain

$$\phi_k = \frac{\upsilon_k w_k^{\gamma}}{\sum_{j=1}^{K}\nu_j w_j^{\gamma}}.$$

Define $D \equiv \sum_{j=1}^{K}\nu_j w_j^{\gamma}$. Then:

$$\varepsilon_{R,w} = \gamma\cdot\frac{\sum_{k=1}^{K}\frac{\bar{w}_k^{\gamma}w^{\gamma}\upsilon_k\bar{w}_k^{\gamma}}{(w^{\gamma}+\bar{w}_k^{\gamma})^2 D}}{\sum_{k=1}^{K}\frac{w^{\gamma}\upsilon_k\bar{w}_k^{\gamma}}{(w^{\gamma}+\bar{w}_k^{\gamma})D}}$$

which is equal to

$$\varepsilon_{R,w} = \gamma\cdot\frac{\sum_{k=1}^{K}\frac{\bar{w}_k^{\gamma}w^{\gamma}\upsilon_k\bar{w}_k^{\gamma}}{(w^{\gamma}+\bar{w}_k^{\gamma})^2}}{\sum_{k=1}^{K}\frac{w^{\gamma}\upsilon_k\bar{w}_k}{(w^{\gamma}+\bar{w}_k^{\gamma})}}$$

Cancel w^γ as there is no k:

$$\varepsilon_{R,w} = \gamma \cdot \frac{\sum_{k=1}^{K} \frac{(\bar{w}_k^\gamma)^2 \upsilon_k}{(w^\gamma + \bar{w}_k^\gamma)^2}}{\sum_{k=1}^{K} \frac{\upsilon_k \bar{w}_k^\gamma}{(w^\gamma + \bar{w}_k^\gamma)}}$$

The condition then becomes

$$\gamma \cdot \frac{\sum_{k=1}^{K} \frac{(\bar{w}_k^\gamma)^2}{(w^\gamma + \bar{w}_k^\gamma)^2} \upsilon_k}{\sum_{k=1}^{K} \frac{\bar{w}_k^\gamma}{(w^\gamma + \bar{w}_k^\gamma)} \upsilon_k} > \gamma \lambda_{EE} f(\theta) \sum_{k=1}^{K} \frac{\bar{w}_k^\gamma}{w^\gamma + \bar{w}_k^\gamma} \upsilon_k.$$

Define the function:

$$f_k = \frac{\bar{w}_k^\gamma}{w^\gamma + \bar{w}_k^\gamma}.$$

Rewriting the condition to obtain

$$\frac{\sum_{k=1}^{K} f_k^2 \upsilon_k}{\sum_{k=1}^{K} f_k \upsilon_k} > \lambda_{EE} f(\theta) \sum_{k=1}^{K} f_k \upsilon_k.$$

which is equal to

$$\sum_{k=1}^{K} f_k^2 \upsilon_k > \lambda_{EE} f(\theta) (\sum_{k=1}^{K} f_k \upsilon_k)^2, \tag{38}$$

which holds by Jensen's inequality as $\lambda_{EE} f(\theta) < 1$.

B.5.4 Proving Uniqueness

Finally, we show that all the endogenous outcomes θ, $\tilde{w}$, N_j, w_j, and V_j can be solved for as functions of parameters. Using the result from Appendix B.5.1 that in any equilibrium firms will have a constant value of $\varepsilon_{R,w} - \varepsilon_{S,w} = \gamma$, and as such optimal wages and employment will take the form of (34) and (35), we then confirm that the stationary equilibrium is unique.

Lemma 3. *Vacancy shares υ_j are a function of only common parameters and do not depend on aggregate wages $\tilde{w}$ or labor market tightness θ.*

Proof: First we want to show that relative wages are not a function of $\tilde{w}$ or θ (assuming a common $\tilde{\alpha}$ across firms). For two firms j and k, relative wages are:

$$\frac{w_j}{w_k} = \left(\frac{c_j}{c_k}\right)^{\frac{1-\tilde{\alpha}}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}} \left(\frac{\tilde{A}_j}{\tilde{A}_k}\right)^{\frac{\sigma}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}}$$

Next we show that relative employment is not a function of $\tilde{w}$ or θ. For two firms j and k, relative employment is:

$$\frac{N_j}{N_k} = \left(\frac{c_j}{c_k}\right)^{\frac{-1}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}} \left(\frac{\tilde{A}_j}{\tilde{A}_k}\right)^{\frac{1+\gamma(1+\chi)}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}}$$

Thus, relative vacancies are a function of parameters only:

$$\frac{V_j}{V_k} = \frac{N_j}{N_k}\left(\frac{w_j}{w_k}\right)^{-\gamma} = \left(\frac{c_j}{c_k}\right)^{\frac{-1-\gamma(1-\tilde{\alpha})}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}} \left(\frac{\tilde{A}_j}{\tilde{A}_k}\right)^{\frac{1+\gamma(1+\chi)-\gamma\sigma}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}}.$$

Let ω_j be the mass of firms with parameters c_j and $\tilde{A}_j$, with $\int_{\tilde{A}}\int_c \omega_j \, dc \, d\tilde{A} = 1$. The share of vacancies of firm type j is

$$\upsilon_j = \frac{\omega_j V_j}{\int_{k \in J} \omega_k V_k} = \frac{1}{\int_{k\in J} \frac{\omega_k}{\omega_j}\frac{V_k}{V_j}}.$$

Since V_j/V_k is a function of only parameters, and the share of each type of firm ω_j is a parameter, then the vacancy share υ_j is a function of only parameters.

Lemma 4. *The ratio of a firm's optimal wage w_j to the index of aggregate wages $\tilde{w}$ is a function of parameters.*

Proof: (assuming a common $\tilde{\alpha}$ across firms)

$$\begin{aligned} \frac{\tilde{w}}{w_j} &= \frac{\left(\int_k \upsilon_k \, w_k^{\gamma} \, dk\right)^{1/\gamma}}{w_j} \\ &= \frac{\left(\int_k \upsilon_k \left[\underbrace{c_k \gamma(1+\chi)(\lambda_{EE}\theta)^{1+\chi}\tilde{w}^{\gamma(1+\chi)}}_{=:X}\right]^{\frac{\gamma(1-\tilde{\alpha})}{D}} \left(\frac{\tilde{\alpha}\tilde{A}_k\gamma(1+\chi)}{1+\gamma(1+\chi)+\sigma}\right)^{\frac{\gamma\sigma}{D}} dk\right)^{1/\gamma}}{\left[X^{\frac{1-\tilde{\alpha}}{D}}\left(\frac{\tilde{\alpha}\tilde{A}_j\gamma(1+\chi)}{1+\gamma(1+\chi)+\sigma}\right)^{\frac{\sigma}{D}}\right]}, \end{aligned}$$

where $D \equiv (1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma$. All factors in X (i.e., γ, χ, λ_{EE}, θ, and the common power of $\tilde{w}$) cancel between the numerator and denominator. Therefore,

$$\frac{\tilde{w}}{w_j} = \frac{\left(\int_k \upsilon_k \, c_k^{\frac{\gamma(1-\tilde{\alpha})}{D}} \, \tilde{A}_k^{\frac{\gamma\sigma}{D}} \, dk\right)^{1/\gamma}}{c_j^{\frac{1-\tilde{\alpha}}{D}} \, \tilde{A}_j^{\frac{\sigma}{D}}}.$$

Because the vacancy shares υ_k are functions only of parameters (see Lemma above), the ratio $\tilde{w}/w_j$ is a function of parameters alone and does not depend on labor-market tightness θ (nor on λ_{EE}, γ, or χ).

Thus, we can write relative wages as a function of parameters only:

$$\frac{\tilde{w}}{w_j} = \frac{\tilde{w}}{w_j}\left(\mathbb{c}, \tilde{\mathbb{A}}\right).$$

Lemma 5. *Given distributions of firm-specific parameters $\tilde{\mathbb{A}}$ and $\mathbb{c}$, the wage index $\tilde{w}$ is an increasing function of tightness θ.*

Proof: Based on equation (34), individual firm wages w_j are increasing in aggregate wages $\tilde{w}$ and tightness θ. Solving out individual wages, and solving $\tilde{w}$ in terms of only θ and parameters yields:

$$\tilde{w} = \Big(\int_j \upsilon_j w_j^\gamma dj\Big)^{\frac{1}{\gamma}}$$

$$\tilde{w} = \Big(\int_j \upsilon_j \Big(\theta^{1+\chi}\tilde{w}^{\gamma(1+\chi)}\Big)^{\frac{\gamma(1-\tilde{\alpha})}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}} g(c_j, \tilde{A}_j) dj\Big)^{\frac{1}{\gamma}}$$

$$\tilde{w} = \Big(\theta^{1+\chi}\tilde{w}^{\gamma(1+\chi)}\Big)^{\frac{(1-\tilde{\alpha})}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}} \Big(\int_j \upsilon_j g(c_j, \tilde{A}_j) dj\Big)^{\frac{1}{\gamma}}, \tag{39}$$

with

$$g(c_j, \tilde{A}_j) = \Big(c_j\gamma(1+\chi)\lambda_{EE}^{1+\chi}\Big)^{\frac{\gamma(1-\tilde{\alpha})}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}} \Big(\frac{\tilde{\alpha}\tilde{A}_j\gamma(1+\chi)}{1+\gamma(1+\chi)+\sigma}\Big)^{\frac{\gamma\sigma}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}}.$$

Grouping all $\tilde{w}$ terms on the left hand side yields:

$$\tilde{w}^{\frac{(1-\tilde{\alpha})+\sigma}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}} = \theta^{\frac{(1+\chi)(1-\tilde{\alpha})}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}} \Big(\int_j \upsilon_j g(c_j, \tilde{A}_j) dj\Big)^{\frac{1}{\gamma}}. \tag{40}$$

Simplifying yields:

$$\tilde{w} = \theta^{\frac{(1+\chi)(1-\tilde{\alpha})}{(1-\tilde{\alpha})+\sigma}} \Big(\int_j \upsilon_j g(c_j, \tilde{A}_j) dj\Big)^{\frac{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}{\gamma(1-\tilde{\alpha}+\sigma)}}.$$

Proof of Proposition Let $\omega(\tilde{A}, c)$ be the joint density function of the mass of firm populations over parameters c and $\tilde{A}$. Then the aggregate mass of vacancies V is

$$V = \int_{\tilde{A}}\int_c V_j \omega(\tilde{A}, c)\, dc\, d\tilde{A}$$

$$V = \int_{\tilde{A}}\int_c \lambda_{EE}\theta\Big(\frac{\tilde{w}}{w_j}\Big)^\gamma N_j \omega(\tilde{A}, c)\, dc\, d\tilde{A}$$

We have already shown that $\tilde{w}/w_j$ is a function of parameters:

$$V = \int_{\tilde{A}}\int_c \lambda_{EE}\theta\Big(\frac{\tilde{w}}{w_j}(\mathbb{c}, \tilde{\mathbb{A}})\Big)^\gamma N_j \omega(\tilde{A}, c)\, dc\, d\tilde{A} \tag{41}$$

We can abbreviate the expression for optimal employment as

$$N_j = \Big(\theta^{1+\chi}\tilde{w}^{\gamma(1+\chi)}\Big)^{\frac{-1}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}} f(c_j, \tilde{A}_j), \tag{42}$$

with

$$f(c_j, \tilde{A}_j) = \Big(c_j \gamma (1+\chi) \lambda_{EE}^{1+\chi}\Big)^{\frac{-1}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}} \Big(\frac{\tilde{\alpha}\tilde{A}_j \gamma (1+\chi)}{1+\gamma(1+\chi)+\sigma}\Big)^{\frac{1+\gamma(1+\chi)}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}}.$$

Plugging in (40) and (42) into equation (41) yields:

$$V = \int\int \theta \times \theta^{\frac{-1-\chi}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}} \times \theta^{\frac{(1+\chi)(1-\tilde{\alpha})}{(1-\tilde{\alpha})+\sigma}\frac{-\gamma(1+\chi)}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}} H(\tilde{A}_j, c_j, \tilde{\mathbb{A}}, \mathbb{c}) dc d\tilde{A},$$

with

$$H(\tilde{A}_j, c_j, \tilde{\mathbb{A}}, \mathbb{c}) = \Big(\frac{\tilde{w}}{w_j}(\mathbb{c}, \tilde{\mathbb{A}})\Big)^{\gamma} f(c_j, \tilde{A}_j)\Big(\int_j \upsilon_j g(c_j, \tilde{A}_j)\, dj\Big)^{\frac{-1-\chi}{(1-\tilde{\alpha})+\sigma}} \omega(\tilde{A}_j, c_j).$$

In equilibrium where all workers are employed, and employed workers search with probability λ_{EE}, then (normalizing the worker population to 1) tightness θ is:

$$\theta = \frac{V}{\lambda_{EE}}.$$

The prior equation then becomes:

$$\lambda_{EE}\theta = \int\int \theta \times \theta^{\frac{-1-\chi}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}} \times \theta^{\frac{(1+\chi)(1-\tilde{\alpha})}{(1-\tilde{\alpha})+\sigma}\frac{-\gamma(1+\chi)}{(1-\tilde{\alpha})(1+\gamma(1+\chi))+\sigma}} H(\tilde{A}_j, c_j, \tilde{\mathbb{A}}, \mathbb{c})\, dc\, d\tilde{A}.$$

The θ terms on the right-hand side can be pulled out of the integral, and a θ term on both sides cancels. With some algebra, we have

$$\theta = \Big(\frac{1}{\lambda_{EE}} \int\int H(\tilde{A}_j, c_j, \tilde{\mathbb{A}}, \mathbb{c})\, dc\, d\tilde{A}\Big)^{\frac{(1-\tilde{\alpha})+\sigma}{1+\chi}}.$$

Thus equilibrium labor market tightness can be calculated strictly as a function of parameters. Plugging in our value of θ in equation (40), we can solve for the aggregate wage $\tilde{w}$ as a function of parameters. Given $\tilde{w}$, we can compute individual firms' N_j, w_j, and V_j.

B.6 Oligopsony

In this section, we derive the first order conditions for the oligopsonistic firm's problem. After deriving the first order conditions, we prove Proposition 2. We solve the firm's problem by first assuming that there is no unemployment, which can be achieved in equilibrium so long as the unemployment benefit b is sufficiently low (under this assumption, $U_t \equiv 0$ so $\Phi^U_{t-1} = 0$ and the unemployment terms in (48)–(49) drop out).

Firms Firms solve

$$\max_{\{V_{j,t}\}_{t=0}^{\infty},\, \{N_{j,t}\}_{t=0}^{\infty},\, \{N_{j,s,t}\}_{\substack{s\le t,\\ t=0}}^{\infty},\, \{w_{j,s,t}\}_{\substack{s\le t,\\ t=0}}^{\infty}} \sum_{t=0}^{\infty} \beta_f^t \Bigg[A_{j,t}\left(K_{j,t}^{\alpha} N_{j,t}^{1-\alpha}\right)^{\frac{\epsilon-1}{\epsilon}} - \sum_{s\le t} w_{j,s,t}\, N_{j,s,t}$$
$$- r^K K_{j,t} - c_j \left(\frac{V_{j,t}}{N_{j,t-1}}\right)^{\chi} N_{j,t-1}^{\sigma} V_{j,t} \Bigg] \tag{43}$$

subject to

$$\text{Recruiting:} \quad N_{j,t,t} = R(\mathcal{V}_{j,t,t}) V_{j,t} \quad \forall t \tag{44}$$

$$\text{Retention:} \quad N_{j,s,t} = (1 - S(\mathcal{V}_{j,s,t})) N_{j,s,t-1} \quad \forall t \text{ and } s < t \tag{45}$$

$$\text{Employment is sum of cohorts:} \quad N_{j,t} = \sum_{s=-\infty}^{t} N_{j,s,t} \quad \forall t \tag{46}$$

$$\text{Wage at least posted wage:} \quad w_{j,s,t} \ge w_{j,s,s} \quad \forall t \text{ and } s < t, \tag{47}$$

where R and S are

$$R\big(\mathcal{V}_{j,t,t}, \mathcal{V}_{-j,t}, V_{j,t}, V_{-j,t}, N_{j,t-1}, N_{-j,t-1}\big) = g(\theta_t)\Big[(1-\Phi^U_{t-1}) \sum_{k\ne j} \sum_{s=-\infty}^{t-1} \phi^n_{k,s,t-1} \frac{\exp\big(\gamma\, \mathcal{V}_{j,t,t}\big)}{\exp\big(\gamma\, \mathcal{V}_{j,t,t}\big) + \exp\big(\gamma\, \mathcal{V}_{k,s,t}\big)}$$
$$+ \Phi^U_{t-1}\, \mathbb{1}\{\mathcal{V}_{j,t,t} > \mathcal{V}_U\}\Big] \tag{48}$$

$$S\big(\mathcal{V}_{j,s,t}, \mathcal{V}_{-j,t}, V_{j,s,t}, V_{-j,t}\big) = \lambda_{EE} f(\theta_t) \sum_{k\ne j} \phi^v_{k,t} \frac{\exp\big(\gamma\, \mathcal{V}_{k,t,t}\big)}{\exp\big(\gamma\, \mathcal{V}_{j,s,t}\big) + \exp\big(\gamma\, \mathcal{V}_{k,t,t}\big)}$$
$$+ \big(1 - \lambda_{EE} f(\theta_t)\big)\, \mathbb{1}\{\mathcal{V}_U > \mathcal{V}_{j,s,t}\} \tag{49}$$

where $\phi^n_{k,s,t-1}$ is the share of workers who were hired in period s by firm k and still employed in period $t-1$, $\phi^v_{k,t}$ is firm k's share of vacancies in period t, and $\Phi^U_{t-1} = \frac{U_{t-1}}{U_{t-1}+\lambda_{EE}(1-U_{t-1})}$ is the share of searchers who are unemployed.

Lagrangian. The Lagrangian is

$$\mathcal{L} \;=\; \sum_{t=0}^{\infty}\beta_f^t\Bigg(A_{j,t}\left(K_{j,t}^{\alpha}\left(\sum_{s=-\infty}^{t}N_{j,s,t}\right)^{1-\alpha}\right)^{\frac{\epsilon-1}{\epsilon}} \;-\; \sum_{s=-\infty}^{t} w_{j,s,t}\,N_{j,s,t} \;-\; c_j V_{j,t}^{1+\chi}\left(\sum_{s=-\infty}^{t-1}N_{j,s,t-1}\right)^{\sigma-\chi}$$

$$-\; r^K K_{j,t} \;+\; \sum_{s=-\infty}^{t-1}\mu_{s,t}\Bigg[-\,N_{j,s,t}+N_{j,s,t-1}\left(1-\lambda_{EE}f(\theta_t)\sum_{k\neq j}\phi_{k,t}^{v}\,\frac{\exp(\gamma\mathcal{V}_{k,t,t})}{\exp(\gamma\mathcal{V}_{j,s,t})+\exp(\gamma\mathcal{V}_{k,t,t})}\right)\Bigg]$$

$$+\;\lambda_t\Bigg[-\,N_{j,t,t}+V_{j,t}\,g(\theta_t)\sum_{k\neq j}\sum_{s=-\infty}^{t-1}\phi_{k,s,t-1}^{n}\,\frac{\exp(\gamma\mathcal{V}_{j,t,t})}{\exp(\gamma\mathcal{V}_{j,t,t})+\exp(\gamma\mathcal{V}_{k,s,t})}\Bigg]\Bigg).$$

We take the first-order condition with respect to $w_{j,t,t}$ under the assumption that condition (47) binds: that is, when the firm raises its posted wage, it raises the wage that the firm pays for as long as the cohort of hired workers is in the firm. For the cohort posted at $t = 0$,

$$\mathcal{L}_{w_{j,0,0}}:\quad -\sum_{t=0}^{\infty}\beta_f^t\,N_{j,0,t} \;+\; \lambda_0\,V_{j,0}\,g(\theta_0)\sum_{k\neq j}\sum_{s=-\infty}^{-1}\phi_{k,s,-1}^{n}\,\gamma\frac{\exp(\gamma\mathcal{V}_{j,0,0})\,\exp(\gamma\mathcal{V}_{k,s,0})}{(\exp(\gamma\mathcal{V}_{j,0,0})+\exp(\gamma\mathcal{V}_{k,s,0}))^2}\,\frac{\partial\mathcal{V}_{j,0,0}}{\partial w_{j,0,0}}$$

$$+\;\sum_{t=1}^{\infty}\beta_f^t\,\mu_{0,t}\,N_{j,0,t-1}\,\lambda_{EE}\,f(\theta_t)\sum_{k\neq j}\phi_{k,t}^{v}\,\gamma\frac{\exp(\gamma\mathcal{V}_{j,0,t})\,\exp(\gamma\mathcal{V}_{k,t,t})}{(\exp(\gamma\mathcal{V}_{j,0,t})+\exp(\gamma\mathcal{V}_{k,t,t}))^2}\,\frac{\partial\mathcal{V}_{j,0,t}}{\partial w_{j,0,0}} \;=\; 0.$$

Notes. (i) Only the $t = 0$ recruiting constraint contributes in the first line (hence λ_0, $V_{j,0}$, θ_0); for $t > 0$, $w_{j,0,0}$ does not enter $\mathcal{V}_{j,t,t}$. (ii) For $t \geq 1$, $w_{j,0,0}$ affects retention via $\mathcal{V}_{j,0,t}$ inside $S(\cdot)$, yielding the second-line terms. (iii) If you impose stationarity, you may replace θ_t with θ throughout.

$$\mathcal{L}_{V_{j,t}}:\quad -\,c_j(1+\chi)\,V_{j,t}^{\chi}\,N_{j,t-1}^{\sigma-\chi}$$

$$-\sum_{s=-\infty}^{t-1}\mu_{s,t}\,N_{j,s,t-1}\,\lambda_{EE}\,\frac{\partial f}{\partial\theta}\,\frac{\partial\theta_t}{\partial V_{j,t}}\sum_{k\neq j}\phi_{k,t}^{v}\,\frac{\exp(\gamma\mathcal{V}_{k,t,t})}{\exp(\gamma\mathcal{V}_{j,s,t})+\exp(\gamma\mathcal{V}_{k,t,t})}$$

$$-\sum_{s=-\infty}^{t-1}\mu_{s,t}\,N_{j,s,t-1}\,\lambda_{EE}\,f(\theta_t)\sum_{k\neq j}\frac{\partial\phi_{k,t}^{v}}{\partial V_{j,t}}\,\frac{\exp(\gamma\mathcal{V}_{k,t,t})}{\exp(\gamma\mathcal{V}_{j,s,t})+\exp(\gamma\mathcal{V}_{k,t,t})}$$

$$+\,\lambda_t\,V_{j,t}\,\frac{\partial g}{\partial\theta}\,\frac{\partial\theta_t}{\partial V_{j,t}}\sum_{k\neq j}\sum_{s=-\infty}^{t-1}\phi_{k,s,t-1}^{n}\,\frac{\exp(\gamma\mathcal{V}_{j,t,t})}{\exp(\gamma\mathcal{V}_{j,t,t})+\exp(\gamma\mathcal{V}_{k,s,t})}$$

$$+\,\lambda_t\,g(\theta_t)\sum_{k\neq j}\sum_{s=-\infty}^{t-1}\phi_{k,s,t-1}^{n}\,\frac{\exp(\gamma\mathcal{V}_{j,t,t})}{\exp(\gamma\mathcal{V}_{j,t,t})+\exp(\gamma\mathcal{V}_{k,s,t})} \;=\; 0.$$

$$\mathcal{L}_{N_{j,t,t}}: \quad \text{MRPL}_t \;-\; w_{j,t,t} \;+\; \beta_f\, c_j\,(\chi-\sigma)\, V_{j,t+1}^{1+\chi}\, N_{j,t}^{\sigma-\chi-1} \;-\; \lambda_t \;+\; \beta_f\, \mu_{t,t+1}\left(1 - S(\mathcal{V}_{j,t,t+1})\right)$$
$$+\; \beta_f\, \lambda_{t+1}\, V_{j,t+1}\, g(\theta_{t+1}) \sum_{k\neq j} \frac{\partial \phi^n_{k,t}}{\partial N_{j,t,t}} \frac{\exp(\gamma \mathcal{V}_{j,t+1,t+1})}{\exp(\gamma \mathcal{V}_{j,t+1,t+1}) + \exp(\gamma \mathcal{V}_{k,t+1,t+1})} \;=\; 0.$$

We will prove the first claim in Proposition 2—that all firms choose the same wage when c_j is common and $\sigma = 0$ or A_j is common —by positing symmetric wages $w_j = w_k = \bar{w}$ for all k and then verifying that a common wage is an equilibrium outcome.

In steady state we have $V/N = S/R$. From wage symmetry ($\mathcal{V}_j = \mathcal{V}_k \; \forall k$), we have that $R_j = \frac{g(\theta)}{2}(1-\phi^n_j)$ and $S_j = \frac{\lambda_{EE} f(\theta)}{2}(1-\phi^v_j)$. Further, under wage symmetry, $\phi^n_j = \phi^v_j \equiv \phi_j$, so S_j/R_j simplifies to :

$$\frac{S_j}{R_j} = \lambda_{EE}\theta$$

for all firms (using $f(\theta)/g(\theta) = \theta$).

With some algebra, it can be shown that the first order condition on wages simplifies to

$$\lambda\gamma\frac{1}{4}\frac{\partial \mathcal{V}_j}{\partial w_j}(1-\phi_j)\left(\frac{V_j}{N_j}g(\theta) + \frac{\lambda_{EE} f(\theta) S_j}{1-\beta_f(1-S_j)}\right) = \frac{S_j}{1-\beta_f(1-S_j)}.$$

Using that $V_j/N_j = S_j/R_j = \lambda_{EE}\theta$, as well as that $g(\theta)\theta = f(\theta)$, this expression simplifies to

$$\lambda\gamma\frac{1}{4}\frac{\partial \mathcal{V}_j}{\partial w_j}(1-\phi_j)\lambda_{EE} f(\theta)\left(1 + \frac{S_j}{1-\beta_f(1-S_j)}\right) = \frac{S_j}{1-\beta_f(1-S_j)}. \tag{50}$$

If wages are equal at all firms, then the first order condition on vacancies becomes

$$c_j(1+\chi)V_j^{\chi} N_j^{\sigma-\chi} = -\lambda\frac{N}{V}\lambda_{EE} f(\theta)\varepsilon_{f,\theta}\varepsilon_{\theta,V_j}(1-\phi_j)\frac{1}{2} - \lambda\frac{N_j}{V_j}\phi_j\lambda_{EE} f(\theta))(\phi_j - 1)\frac{1}{2}$$
$$+\lambda g(\theta)\varepsilon_{g,\theta}\varepsilon_{\theta,V_j}(1-\phi_j)\frac{1}{2} + \lambda g(\theta))(1-\phi_j)\frac{1}{2}$$

Combining terms and using that $V/N = \lambda_{EE}\theta$, we therefore have $\frac{N}{V} = \frac{R}{S} = \frac{1}{\lambda_{EE}\theta}$, and $\varepsilon_{f,\theta} = 1 + \varepsilon_{g,\theta}$ (from the matching function being constant returns to scale), the first order condition on vacancies simplifies down to

$$c_j(1+\chi)V^{\chi} N^{\sigma-\chi} = \lambda\frac{1}{2}(1-\phi_j)g(\theta) \tag{51}$$

Plugging this in to (50) yields

$$c_j(1+\chi)V^{\chi} N^{\sigma-\chi}\gamma\frac{1}{2}\frac{\partial \mathcal{V}_j}{\partial w_j}\lambda_{EE}\theta\left(1 + \frac{S_j}{1-\beta_f(1-S_j)}\right) = \frac{S_j}{1-\beta_f(1-S_j)} \tag{52}$$

Workers The worker gets utility from wages and draws a utility shock when given a choice between two firms. Let $\phi^v_{k,t}$ be the share of vacancies posted by firm k in period t. If workers encounter their own firm's vacancies, the workers redraw preferences (as if they are choosing between two jobs), but the firm is not obligated to pay the same wage as its other hires in period t, and the worker continues to receive the wage of the cohort of workers they were initially hired with in period s.

$$\begin{aligned}\mathcal{V}_{j,s,t} = &\frac{\eta}{\eta-1}\left(w_{j,s,t}^{\frac{\eta-1}{\eta}}-1\right)+\beta_w s_0 \mathcal{V}_U + \\ &\beta_w(1-s_0)\mathbb{E}_t\Bigg[\lambda_{EE} f(\theta)\Bigg(\sum_{k\neq j}\phi^v_{k,t}\gamma^{-1}\log\Bigg(\exp\left(\gamma\mathcal{V}_{j,s,t+1}\right)+\exp\left(\gamma\mathcal{V}_{k,t+1}\right)\Bigg)+ \\ &\phi^v_{j,t}\gamma^{-1}\log\Bigg(\exp(\gamma\mathcal{V}_{j,s,t+1})+\exp(\gamma\mathcal{V}_{j,s,t+1})\Bigg)\Bigg)+(1-\lambda_{EE} f(\theta))\max\{\mathcal{V}_{j,t+1,t+1},\mathcal{V}_U\}\Bigg],\end{aligned} \tag{53}$$

where the second logarithm compares staying in the current job (value $\mathcal{V}_{j,s,t+1}$) with accepting a new vacancy at firm j (value $\mathcal{V}_{j,t+1,t+1}$)

We will now consider one-period deviations in period t for firms hiring workers in period t, under the assumption from workers that the posted wage will be equal to their wage for the duration of their match with the firm.

$$\frac{\partial\mathcal{V}_{j,t,t}}{\partial w_{j,t,t}} = w_{j,t,t}^{-\frac{1}{\eta}}+\beta_w\Bigg(f(\theta)\lambda_{EE}\Bigg(\sum_{k\neq j}\phi^v_{k,t}\left(\frac{\exp\left(\gamma\mathcal{V}_j\right)}{\exp\left(\gamma\mathcal{V}_j\right)+\exp\left(\gamma\mathcal{V}_k\right)}\right)+\phi^v_{j,t}\Bigg)+\Big(1-f(\theta)\lambda_{EE}\Big)\Bigg)\frac{\partial\mathcal{V}_j}{\partial w_j}.$$

We can substitute in the expression for the separation rate from equation (3) (under the assumption that workers are not quitting into unemployment or exogenously separating). Under symmetry, wages are equal across firms ($w_j = w_k$), so $\mathcal{V}_j = \mathcal{V}_k$, and employment and vacancy shares coincide ($\phi^n_{k,t} = \phi^v_{k,t}$). If there are no exogenous separations ($s_0 = 0$), then $S_j = \lambda_{EE}\, f(\theta)\,(1-\phi^v_{j,t})/2$.

$$\begin{aligned}\frac{\partial\mathcal{V}_j}{\partial w_j} =& w_j^{-\frac{1}{\eta}}+\beta_w\Bigg(f(\theta)\lambda_{EE}\left(\frac{(1-\phi^v_{jt})}{2}+\phi^v_{j,t}\right)+\Big(1-f(\theta)\lambda_{EE}\Big)\Bigg)\frac{\partial\mathcal{V}_j}{\partial w_j} \\ =& w_j^{-\frac{1}{\eta}}+\beta_w\Bigg(1-\frac{\lambda_{EE} f(\theta)}{2}(1-\phi^v_{j,t})\Bigg)\frac{\partial\mathcal{V}_j}{\partial w_j} \\ =& w_j^{\frac{-1}{\eta}}+\beta_w\Big(1-S_j\Big)\frac{\partial\mathcal{V}_j}{\partial w_j},\end{aligned}$$

where the last line uses that $S_j = \lambda_{EE} f(\theta)/2 \times (1 - \phi^v_{j,t})$ under symmetry. Thus

$$\frac{\partial \mathcal{V}_j}{\partial w_j} = \frac{w_j^{-\frac{1}{\eta}}}{(1 - \beta_w(1 - S_j))}. \tag{54}$$

Combining Elements Plugging in our equation (54), the first order condition on wages further simplifies to

$$c_j(1+\chi)V_j^{\chi} N_j^{\sigma-\chi}\gamma\frac{1}{2}\lambda_{EE}\theta\frac{w_j^{-\frac{1}{\eta}}}{(1-\beta_w(1-S_j))} = \left(1 + \frac{S_j}{1-\beta_f(1-S_j)}\right) = \frac{S_j}{1-\beta_f(1-S_j)} \tag{55}$$

As $\beta_f \to 1$ and $\beta_w \to 0$, this becomes

$$c_j(1+\chi)V_j^{\chi} N_j^{\sigma-\chi}\gamma\lambda_{EE}\theta w_j^{-\frac{1}{\eta}} = 1 \tag{56}$$

Recalling that $\lambda_{EE}\theta = S/R = V/N$, this expression becomes

$$\gamma c_j(1+\chi)(\lambda_{EE}\theta)^{1+\chi} N_j^{\sigma} = w_j^{\frac{1}{\eta}}. \tag{57}$$

Thus if $\sigma = 0$ and c_j are common among firms, then the wage is identical for all firms, regardless of their value of A_j and their size N_j, proving the first claim in Proposition 2. Additionally, if firms are ex-ante identical (common values of c_j and A_j), they will choose identical wages.

Turning to employment, if all firms pay the same wage, the first order condition on employment simplifies to

$$MRPL_j - w_j + c_j(\chi-\sigma)V_j^{1+\chi}N_j^{\sigma-\chi-1} - \lambda S_j - \lambda V_j g(\theta)\frac{1}{2} = 0$$

Taking advantage of the fact that $N_j = \phi_j$ ((normalizing total employment to 1) and $R = \frac{g(\theta)}{2}(1-\phi_j)$, with some algebra this expression simplifies to

$$MRPL_j - w_j + c_j(\chi-\sigma)V_j^{1+\chi}N_j^{\sigma-\chi-1} - \lambda\frac{S_j}{1-\phi_j} = 0. \tag{58}$$

Combining (58) with (51) and (57), as well as using that $R_j = \frac{g(\theta)}{2}(1-\phi_j)$ under wage symmetry, $V_j/N_j = \lambda_{EE}\theta \ \forall j$, and $\phi^n_j = \phi^v_j$, with some algebra and making workers have log utility ($\eta \to 1$), yields our expression for the firm's recruiting cost-adjusted markdown

$$\frac{w_j + c_j\left(\frac{V_j}{N_j}\right)^{1+\chi}N_j^{\sigma}}{MRPL_j} = \frac{1+\gamma(1+\chi)}{1+\gamma(1+\chi)+\sigma+(1+\chi)\frac{\phi^n_j}{1-\phi^n_j}}, \tag{59}$$

establishing the second claim of Proposition 2.

B.7 Bounds on Markdowns with Oligopsony

In this section, we prove that under two allocations that deliver the same labor market HHI, the aggregate recruiting cost-adjusted markdown $\mathcal{M}$ is the lowest when that HHI is achieved with one large firm. Let ϕ_j^n be firm j's share of employment. The employment HHI is $HHI = \sum_j (\phi_j^n)^2$.

Consider two hypothetical allocations: one in which a given HHI is achieved by one firm: $(\phi_1^n)^2 = \overline{HHI}$, where all other firms are sufficiently small such that they do not contribute to the HHI. Consider a second allocation in which two firms achieve that same HHI: $(\phi_2^n)^2 + (\phi_3^n)^2 = \overline{HHI}$, and all other firms are sufficiently small. This gives $(\phi_1^n)^2 = (\phi_2^n)^2 + (\phi_3^n)^2$.

Without loss of generality, we can assume $\phi_3^n \leq \phi_2^n$. The aggregate markdown is

$$\mathcal{M} = \frac{1}{\sum_j \phi_j^n/\mu_j}, \qquad \text{with} \qquad \mu_j = \frac{1+\gamma(1+\chi)}{1+\gamma(1+\chi)+\sigma+(1+\chi)\frac{\phi_j^n}{1-\phi_j^n}}$$

Thus for a given $\overline{HHI}$ and ϕ_2^n, and focusing on the case of $\sigma = 0$, the aggregate markdown can be written as

$$\mathcal{M} = \frac{1+\gamma(1+\chi)}{1+\gamma(1+\chi)+(1+\chi)\left(\frac{(\phi_2^n)^2}{1-\phi_2^n}+\frac{(\phi_1^n)^2-(\phi_2^n)^2}{1-((\phi_1^n)^2-(\phi_2^n)^2)^{.5}}\right)}$$

The goal is to show that this expression is decreasing in ϕ_2^n. Define

$$A(\phi_2^n) = \frac{(\phi_2^n)^2}{1-\phi_2^n} + \frac{(\phi_1^n)^2-(\phi_2^n)^2}{1-((\phi_1^n)^2-(\phi_2^n)^2)^{.5}}.$$

Then we just need to show that $\mathcal{M}$ is a decreasing function of ϕ_2^n if A is increasing in ϕ_2^n. With some algebra, we have

$$\frac{\partial A(\phi_2^n)}{\partial \phi_2^n} = \phi_2^n\left(\frac{(2-\phi_2^n)}{(1-\phi_2^n)^2} - \frac{(2-\phi_3^n)}{(1-\phi_3^n)^2}\right)$$

The function $\frac{2-x}{(1-x)^2}$ is increasing for $x \in [0,1)$, so since $\phi_3^n \leq \phi_2^n$ we have $\frac{\partial A}{\partial \phi_2^n} > 0$. Thus $\mathcal{M}$ is decreasing in ϕ_2^n (for fixed $\overline{HHI}$), and is therefore minimized when ϕ_2^n is as large as possible—i.e., when the given HHI is achieved by a single large firm.

C Supplemental Materials

C.1 Calibration with Forward-Looking Workers

In Section 2.7, we assumed that workers are myopic $\beta_w = 0$ and that there are no exogenous separations $s_0 = 0$, as this delivered a constant sum of recruiting and separation elasticities. In this section, we examine how the model changes if we choose a more realistic calibration. We now set workers' discount factor $\beta_w = .99$, and we set the exogenous separation rate $s = 0.025$ to get a steady-state unemployment rate between 4 and 5 percent. We assume firms are ex-ante identical. Define $\xi_{R,w}$ as the elasticity of the recruiting elasticity with respect to a firm's wage policy, and similarly $\xi_{S,w}$ as the elasticity of the separation elasticity with respect to a firm's wage policy.

Table 11: Calibration with Forward Looking Workers

Parameter		Value	Reason
γ	Inverse scale parameter of non-wage preferences	1/4.3	Match $\varepsilon_R - \varepsilon_S = 4$
η	Worker CRRA parameter	.36	$\xi_{R,w} \approx \xi_{S,w}$
s_0	Exogenous separation rate	2.5	Match $U = 4.3$
c	Hiring cost constant	625	Match vacancy rate
χ	Hiring cost-vacancy rate elasticity	2	Evidence in Section 3
σ	Hiring cost-size elasticity	0	Evidence in Section 3
λ_{EE}	On-the-job search probability	.14	Match J-J mobility
b	Unemployment benefit	.1	Workers accept low wage offers

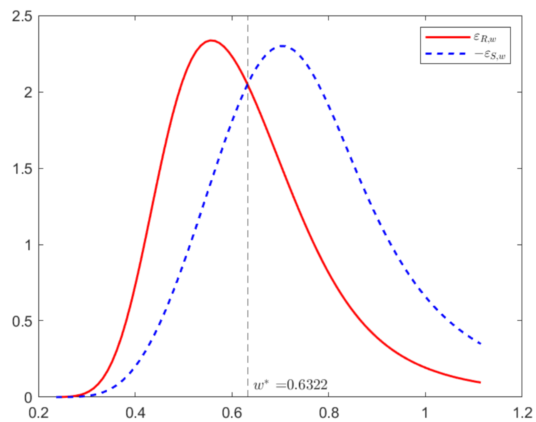
The equilibrium wage is 0.6322. Figure 7 plots what the recruiting and separation rates would be if a firm chose alternative wages. Consistent with evidence from Hirsch et al. (2022) and Langella and Manning (2021), the recruiting and separation elasticities are highest in the middle of the wage distribution. Because some separations are exogenous (and therefore are not wage sensitive), the separation elasticity declines at high wages because the only separations are exogenous and marginal changes in wages do not affect separation probabilities very much. At low wages, the separation elasticity is low because workers leave as soon as they find another job, and marginal changes in the wage have little effect on that separation decision. The recruiting elasticity is low at low wages for similar reasons: the only workers who accept jobs are the wage-insensitive unemployed workers. At high wages, the recruiting elasticity is low because all workers will accept the job if they find it, and marginal changes in wages have little effect.

The key difference in calibration is the value of γ: as workers become more forward-looking, the variance of non-wage preference draws can be rescaled to maintain $\varepsilon_{R,w} - \varepsilon_{S,w}$

around 4. For this calibration, we set $\gamma = 0.23$.

The other difference in calibration is η, the CRRA preference parameter in worker's flow utility. This is calibrated to offset the option value that would arise if workers have log utility and are forward-looking. With log utility, the feature of workers being forward-looking disproportionately lowers recruiting and separation elasticities at low wage levels. This is because of option value: workers know they will not be in low-wage jobs as long, and so marginal changes in wages have smaller effects on workers' value functions at low wages compared to at high wages. This feature makes recruiting and separation elasticities rise with wages, all else equal. This can be offset by making workers more risk averse: with a lower value of η, workers care about marginal increases of wages less as wages go up.

Figure 7: Recruiting and Separation Elasticities for Different Wage Levels β^w=.99, s_0=.025



This figure plots the recruiting elasticity $\varepsilon_{R,w}$ and separation elasticity $\varepsilon_{S,w}$ for different wages if all other firms pay the equilibrium wage of $w = 0.6322$. Both the recruiting and separation elasticities peak near the equilibrium wage and become small for high and low values of the wage.

Our estimation of χ and σ are not affected by workers being myopic, as long as other parameters (namely η and γ) are appropriately calibrated. The general form of the log-linearized equation is

$$\hat{w}_t = \frac{1}{\Omega}\Bigg(\frac{\varepsilon_{R,w}}{\varepsilon_{R,w} - \varepsilon_{S,w}}(\chi\hat{h}_t + \sigma\chi\hat{N}_{t-1}) + \frac{-\varepsilon_{S,w}}{\varepsilon_{R,w} - \varepsilon_{S,w}}\left(S\sum_{\kappa=0}^{\infty}(1-S)^{\kappa}(-(1+\chi)\varepsilon_{R,w}\hat{w}_{t+\kappa+1} + \chi\hat{h}_{t+\kappa+1} + \sigma\chi\hat{N}_{t+\kappa})\right)\Bigg) \tag{60}$$

where $\Omega = 1 + \frac{\varepsilon_{R,w}}{\varepsilon_{R,w}-\varepsilon_{S,w}}(-\xi_{R,w} - \varepsilon_{S,w} + (1+\chi)\varepsilon_{R,w}) + \frac{-\varepsilon_{S,w}}{\varepsilon_{R,w}-\varepsilon_{S,w}}(-\xi_{S,w} - \varepsilon_{S,w})$ Under our calibration above, estimating σ and χ using (60), using the procedure outlined in Section 3.4 around a steady state with no wage dispersion, would yield (approximately) the same parameter estimates as in the main text.

C.2 Extension with Worker Heterogeneity

In this section, we show that we can extend the model in Section 2.7 where workers differ in ability and firms pay wages per efficiency unit of labor. We show that the law of motion for the firm is identical up to a constant as in Section 2.7. Wages are therefore log-additive in worker types and firm pay policies, motivating the AKM robustness check in we use in Section C.2.

Suppose that workers have immutable ability a_i. Let the number of workers at a firm be $N_j = \int_i m_j(a_i)di$, and the total amount of labor be $L_j = \int_i m_j(a_i)a_i di$, where $m_j(a_i)$ is the density of ability of workers employed at firm j. Let $h(a_i)$ be the probability density of searchers of ability a_i, so $\int_i h(a_i)di = 1$.

Firms post vacancies as usual, and firms and workers engage in random search. Firms choose efficiency wage policies ω_j, so the amount paid to a worker of type a_i is $w_{ij} = a_i\omega_j$. Firms cannot direct search towards workers of different abilities. If a worker employed at firm j earning wage rate ω_j meets a firm k offering ω_k, the probability that the worker will leave is

$$s(\omega_j, \omega_k, a_i) = \frac{(a_i\omega_k)^\gamma}{(a_i\omega_j)^\gamma + (a_i\omega_k)^\gamma} = \frac{\omega_k^\gamma}{\omega_j^\gamma + \omega_k^\gamma}.$$

The separation rate $\mathbf{S}$ is the share of effective labor that leaves the firm each period.

$$\mathbf{S}(\omega_j) = \frac{\int_i \int_k m_j(a_i) f(\theta) \frac{(a_i\omega_k)^\gamma}{(a_i\omega_j)^\gamma + (a_i\omega_k)^\gamma} \upsilon(\omega_k) dk di}{\int_i m_j(a_i)di}$$

$$\mathbf{S}(\omega_j) = \frac{1}{N_j} \int_i \int_k m_j(a_i) f(\theta) \frac{\omega_k^\gamma}{\omega_j^\gamma + \omega_k^\gamma} \upsilon(\omega_k) dk di$$

$$\mathbf{S}(\omega_j) = \frac{1}{N_j} f(\theta) \int_k \frac{\omega_k^\gamma}{\omega_j^\gamma + \omega_k^\gamma} \upsilon(\omega_k) dk \int_i m_j(a_i) di$$

$$\mathbf{S}(\omega_j) = f(\theta) \int_k \frac{\omega_k^\gamma}{\omega_j^\gamma + \omega_k^\gamma} \upsilon(\omega_k) dk.$$

Thus the separation rate $\mathbf{S}$ is just a function of the firm's wage policy. The rate at which the firm acquires effective labor is

$$\mathbf{H}(V_j, \omega_j) = V_j g(\theta) \int_i \int_k \frac{(a_i\omega_j)^\gamma}{(a_i\omega_j)^\gamma + (a_i\omega_k)^\gamma} \phi(\omega_k, a_i) dk di$$

where $\phi(\omega_k, a_i)$ is the joint distribution of wage policies and abilities of searchers. Let the recruiting rate $\mathbf{R}(\omega_j)$ be the rate at which vacancies are converted into labor.

$$\mathbf{R}(\omega_j) \equiv \frac{H(V_j, \omega_j)}{V_j} = g(\theta) \int_i \int_k \frac{(a_i\omega_j)^\gamma}{(a_i\omega_j)^\gamma + (a_i\omega_k)^\gamma} \phi(\omega_k, a_i) dk di$$

Suppose a candidate equilibrium where the distribution of wage policies is the same across worker types i. Then we can rewrite the recruiting rate as

$$\mathbf{R}(\omega_j) = g(\theta)\int_i\int_k \frac{(a_i\omega_j)^\gamma}{(a_i\omega_j)^\gamma + (a_i\omega_k)^\gamma}\phi(\omega_k)h(a_i)dkdi$$
$$\mathbf{R}(\omega_j) = g(\theta)\int_k \frac{(a_i\omega_j)^\gamma}{(a_i\omega_j)^\gamma + (a_i\omega_k)^\gamma}\phi(\omega_k)dk\int_i h(a_i)di\mathbf{R}(\omega_j) \quad = g(\theta)\int_k \frac{(a_i\omega_j)^\gamma}{(a_i\omega_j)^\gamma + (a_i\omega_k)^\gamma}\phi(\omega_k)dk.$$

The last thing to show is that in equilibrium, the distribution of employed wages $\phi(\omega_k)$ is constant for all worker types. Since the on-the-job search parameter for each worker type is identical, and since the flow rates across firm wage levels are the same, then the distribution of wage levels in equilibrium for each ability type a will be identical. As a result, the density of employment $m_j(a_i)/N_j = h(a_i)\ \forall i$ and j. As a result, the law of motion of effective labor at the firm follows the same form as when there is no worker heterogeneity.

$$L_{j,t} = (1 - \mathbf{S}(\omega_{j,t}))L_{j,t-1} + \mathbf{R}(\omega_{j,t})V_t.$$

Since all firms will have the same distribution of worker types a_i, so $m_j(a_i) = m(a_i)\forall j$, we can simply take the average $L_j = \int_i m_j(a_i)a_i di = \int_i m(a_i)a_i di = \frac{\int_i m(a_i)a_i di}{N_j}N_j = \bar{a}N_j$. Since $\mathbf{S}$ is the share of labor that is lost to turnover each period, and since worker quality will be the same at each firm in every period, we simply have that $\mathbf{S} = S$. As for recruiting, $\mathbf{R}$ is rate of converting vacancies into effective labor, while R is the rate of converting vacancies into the number of workers. Since the average quality of workers will be the same in each period, then $\mathbf{R} = R\bar{a}$. With some algebra, we have

$$\begin{aligned}
L_{j,t} &= (1 - \mathbf{S}(\omega_{j,t}))L_{j,t-1} + \mathbf{R}(\omega_{j,t})V_t \\
N_{j,t}\bar{a} &= (1 - S(\omega_{j,t}))N_{j,t-1}\bar{a} + R(\omega_{j,t})\bar{a}V_t \\
N_{j,t} &= (1 - S(\omega_{j,t}))N_{j,t-1} + R(\omega_{j,t})V_t,
\end{aligned}$$

which is the same law of motion as in the original text except in wage policies ω_j rather than wage levels w_j.

Because the payments to workers are $w_{ij} = a_i\omega_j$, we can simply write log wages as $\log w_{ij} = \log a_i + \log\omega_j$, so the worker effect $\zeta_i = \log a_i$, and the firm effect $\psi_j = \log\omega_j$. For workers who switch, the change in wages $\log w_{ij} - \log_{ik} = \zeta_i + \omega_j - (\zeta_i + \omega_k) = \omega_j - \omega_k$.

C.3 Oligopsony with Hiring Costs

In this section, we solve a general model oligopsony and hiring costs. We abstract from capital and downward sloping product demand, assuming that output = $AN^{1-\alpha}$. Hires each period are $H_t = V_t R(w_t)$. We also assume for simplicity that firms are committed to pay their posted wage, unlike in the main text where the posted wage is a floor. Firms maximize

$$\max_{\{\{N_{t,t+\tau}\}\},\{w_{t+\tau}\},\{V_{t+\tau}\}} \sum_{\tau=0}^{\infty} \beta_f^\tau \Bigg(A\left(\sum_{s=-\infty}^{t+\tau} N_{s,t+\tau}\right)^{1-\alpha} - \sum_{s=-\infty}^{t+\tau} w_s N_{s,t+\tau} - c\left(\frac{V_{t+\tau}}{N_{t+\tau-1}}\right)^{\chi} V_{t+\tau} N_{t+\tau-1}^{\sigma} - c_h \left(\frac{H_{t+\tau}}{N_{t+\tau-1}}\right)^{\chi_h} H_{t+\tau}^{\chi_h} N_{t+\tau-1}^{\sigma_h} \Bigg)$$

subject to

$$\begin{aligned} N_{s,t+\tau} &= (1 - S(w_s)) N_{s,t+\tau-1} \\ N_{t,t} &= R(w_t) V_t, \end{aligned}$$

where

$$S(w_{j,t}, V_{j,t}, w_{-j,t}, V_{-j,t}) = \lambda_{EE} f(\theta(V_{j,t})) \sum_{k \neq j} \phi_{k,t}^{v} \frac{\exp(\gamma \mathcal{V}_{k,t})}{\exp(\gamma \mathcal{V}_{j,t}) + \exp(\gamma \mathcal{V}_{k,t})} \quad (61)$$

$$R(w_{j,t}, V_{j,t}, w_{-j,t}, V_{-j,t}, N_{j,t-1}, N_{-j,t-1}) = g(\theta(V_{j,t})) \sum_{k \neq j} \phi_{k,t-1}^{n} \frac{\exp(\gamma \mathcal{V}_{j,t})}{\exp(\gamma \mathcal{V}_{j,t}) + \exp(\gamma \mathcal{V}_{k,t})} \quad (62)$$

where $\phi_{k,t-1}^{n}$ is the share of workers who are employed at wage w_k the period $t-1$.

The lagrangian is

$$\begin{aligned} \mathcal{L} : \sum_{\tau=0}^{\infty} \beta_f^\tau \Bigg(& A\left(\sum_{s=-\infty}^{t+\tau} N_{s,t+\tau}\right)^{1-\alpha} - \sum_{s=-\infty}^{t+\tau} w_s N_{s,t+\tau} - c\left(\frac{V_{t+\tau}}{N_{t-1+\tau}}\right)^{\chi} V_{t+\tau} N_{t-1+\tau}^{\sigma} - c_h \left(\frac{V_{t+\tau}}{N_{t+\tau-1}}\right)^{\chi_h} V_{t+\tau} N_{t+\tau-1}^{\sigma_h} \\ & \times \left[g(\theta(V_{j,t})) \sum_{k \neq j} \phi_{k,t-1}^{n} \frac{\exp(\gamma \mathcal{V}_{j,t})}{\exp(\gamma \mathcal{V}_{j,t}) + \exp(\gamma \mathcal{V}_{k,t})} \right]^{1+\chi_h} \\ & + \sum_{s=-\infty}^{t+\tau-1} \mu_{s,t+\tau} \left[- N_{s,t+\tau} + N_{s,t+\tau-1} \left(1 - \lambda_{EE} f(\theta(V_{j,t})) \sum_{k \neq j} \phi_{k,t+\tau}^{v} \frac{\exp(\gamma \mathcal{V}_{k,t})}{\exp(\gamma \mathcal{V}_{j,t}) + \exp(\gamma \mathcal{V}_{k,t})} \right) \right] \\ & + \lambda_{t+\tau} \left[- N_{t+\tau,t+\tau} + V_{t+\tau} g(\theta(V_{j,t}))(1 - N_{t+\tau-1}) \frac{\exp(\gamma \mathcal{V}_{j,t})}{\exp(\gamma \mathcal{V}_{j,t}) + \exp(\gamma \mathcal{V}_{k,t})} \right] \Bigg). \end{aligned}$$

FOC on Wages The first order condition on w_t (i.e., $\tau = 0$)

$$\begin{aligned}
\mathcal{L}_{w_{j,t}} :& -\sum_{\tau=0}^{\infty}\beta_f^{\tau}N_{t,t+\tau} + \lambda_t V_t g(\theta_t)(1-N_{j,t-1})\gamma\frac{\exp\left(\gamma\mathcal{V}_{j,t}\right)\exp\left(\gamma\mathcal{V}_{k,t}\right)}{\left(\exp\left(\gamma\mathcal{V}_{j,t}\right)+\exp\left(\gamma\mathcal{V}_{k,t}\right)\right)^2}\frac{\partial\mathcal{V}_{j,t}}{\partial w_{j,t}} \\
&+\sum_{\tau=1}^{\infty}\left(\beta_f^{\tau}\mu_{t,t+\tau}N_{t,t+\tau-1}\lambda_{EE}f(\theta_t)\sum_{k\neq j}\phi^v_{k,t+\tau}\gamma\frac{\exp\left(\gamma\mathcal{V}_{j,t}\right)\exp\left(\gamma\mathcal{V}_{k,t}\right)}{\left(\exp\left(\gamma\mathcal{V}_{j,t}\right)+\exp\left(\gamma\mathcal{V}_{k,t}\right)\right)^2}\frac{\partial\mathcal{V}_{j,t}}{\partial w_{j,t}}\right) \\
&- c_h(1+\chi_h)\gamma g(\theta_t)\left(g(\theta_t)\sum_{k\neq j}\phi^n_{k,t-1}\frac{\exp(\gamma\mathcal{V}_{j,t})}{\exp(\gamma\mathcal{V}_{j,t})+\exp(\gamma\mathcal{V}_{k,t})}\right)^{\chi_h} \\
&\times\sum_{k\neq j}\phi^n_{k,t-1}\frac{\exp(\gamma\mathcal{V}_{j,t})\exp(\gamma\mathcal{V}_{k,t})}{\left(\exp(\gamma\mathcal{V}_{j,t})+\exp(\gamma\mathcal{V}_{k,t})\right)^2}V_t^{\chi_h+1}N_{t-1}^{\sigma_h-\chi_h}\frac{\partial\mathcal{V}_{j,t}}{\partial w_{j,t}} = 0
\end{aligned}$$

Under symmetry, this simplifies to

$$\begin{aligned}
\mathcal{L}_{w_{j,t}} :& -\sum_{\tau=0}^{\infty}\beta_f^{\tau}H(1-S(w_j))^{\tau} + \frac{1}{4}\lambda\gamma(1-\phi_j)\left(Vg(\theta)+\lambda_{EE}f(\theta)\sum_{\tau=1}^{\infty}\beta_f^{\tau}(1-S(w_j))^{\tau}H\right)\frac{\partial\mathcal{V}_j}{\partial w_j} \\
&-\frac{1}{2}c_h(1+\chi_h)\gamma\left(\frac{g(\theta)(1-\phi_j)}{2}\right)^{1+\chi_h}V^{1+\chi_h}N^{\sigma_h-\chi_h}\frac{\partial\mathcal{V}_j}{\partial w_j} = 0
\end{aligned}$$

Taking $\beta_f \to 1$ and replacing V/N with $S/R = \lambda_{EE}\theta$, then plugging in our value for $\frac{\partial\mathcal{V}_j}{\partial w_j}$, we have:

$$\frac{1}{2w_j(1-\beta_w(1-S))}\gamma\lambda_{EE}f(\theta)(1-\phi_j)\left[\lambda_t - \frac{1}{2}c_h(1+\chi_h)N^{\sigma_h}\left(\frac{\lambda_{EE}\ f(\theta)(1-\phi_j)}{2}\right)^{\chi_h}\right] = 1$$

FOC on Vacancies

$$\begin{aligned}
\mathcal{L}_{V_t} : -c(1+\chi)V_t^{\chi}N_{t-1}^{-\chi(1-\sigma)} & \sum_{s=-\infty}^{t} -\lambda_t N_{s,t-1}\lambda_{EE}\frac{\partial f}{\partial\theta}\frac{f}{f}\frac{\theta_t}{\theta_t}\frac{\partial\theta_t}{\partial V_{j,t}}\frac{V_{j,t}}{V_{j,t}}\sum_{k\neq j}\phi^v_{k,t}\frac{\exp\left(\gamma\mathcal{V}_{k,t}\right)}{\exp(\gamma\mathcal{V}_{j,t})+\exp\left(\gamma\mathcal{V}_{k,t}\right)} \\
& \sum_{s=-\infty}^{t} -\lambda_t N_{s,t-1}\lambda_{EE}f(\theta_t(V_{jt}))\sum_{k\neq j}\frac{-\phi^v_{k,t}}{\bar{V}_t}\frac{\exp\left(\gamma\mathcal{V}_{k,t}\right)}{\exp(\gamma\mathcal{V}_{j,t})+\exp\left(\gamma\mathcal{V}_{k,t}\right)} \\
& + \lambda_t V_t\frac{\partial g}{\partial\theta}\frac{g}{g}\frac{\theta_t}{\theta_t}\frac{\partial\theta_t}{\partial V_{j,t}}\sum_{k\neq j}\phi^n_{k,t-1}\frac{\exp(\gamma\mathcal{V}_{j,t})}{\exp(\gamma\mathcal{V}_{j,t})+\exp\left(\gamma\mathcal{V}_{k,t}\right)} \\
& + \lambda_t g(\theta_t(V_{j,t}))\sum_{k\neq j}\phi^n_{k,t-1}\frac{\exp(\gamma\mathcal{V}_{j,t})}{\exp(\gamma\mathcal{V}_{j,t})+\exp\left(\gamma\mathcal{V}_{k,t}\right)} \\
& - c_h(1+\chi_h)V_t^{\chi_h}N_t^{\sigma_h-\chi_h}(g(\theta))^{\chi_h}\left(\sum_{k\neq j}\phi^n_{k,t-1}\frac{\exp(\gamma\mathcal{V}_{j,t})}{\exp(\gamma\mathcal{V}_{j,t})+\exp(\gamma\mathcal{V}_{k,t})}\right)^{1+\chi_h}
\end{aligned}$$

$$\times \left(g(\theta) + V_t \frac{\partial g}{\partial \theta} \frac{\partial \theta_t}{\partial V_{j,t}} \right) = 0.$$

There is a backward looking sum here because a firm posting more vacancies affects the separation rate of all workers who are still in the firm, regardless of what period those workers arrived. Under symmetry, then

$$\begin{aligned}
c(1+\chi)V_j^{\chi} N_j^{-\chi(1-\sigma)} = & -\lambda \frac{N}{V} \lambda_{EE} f \varepsilon_{f,\theta} \varepsilon_{\theta,V_j} (1-\phi_j)\frac{1}{2} \\
& - \lambda \frac{N_j}{V_j} \phi_j \lambda_{EE} f(\theta_t(V_j))(\phi_j - 1)\frac{1}{2} \\
& + \lambda g(\theta) \varepsilon_{g,\theta} \varepsilon_{\theta,V_j} (1-\phi_j)\frac{1}{2} \\
& + \lambda_t g(\theta_t(V_j))(1-\phi_j)\frac{1}{2} \\
& - c_h(1+\chi_h) V_t^{\chi_h} N_t^{\sigma_h - \chi_h} \left(\frac{g(\theta)(1-\phi_j)}{2} \right)^{1+\chi_h} \left(1 + \varepsilon_{g,\theta} \varepsilon_{\theta,V_j} \right)
\end{aligned}$$

Combining terms and using that $V/N = \lambda_{EE}\theta$, we therefore have $\frac{N}{V} = \frac{1}{\lambda_{EE}\theta}$, and $\varepsilon_{f,\theta} = 1 + \varepsilon_{g,\theta}$

$$c(1+\chi)V^{\chi} N^{-\chi(1-\sigma)} = \frac{1}{2} \lambda g(\theta)(1-\phi_j) - c_h(1+\chi_h) V_t^{\chi_h} N_t^{\sigma_h - \chi_h} \left(\frac{g(\theta)(1-\phi_j)}{2} \right)^{1+\chi_h} (1 + \phi_j \varepsilon_{g,\theta})$$

Combining with the FOC on wages and solving for λ_t yields

$$\lambda_t = \frac{2c(1+\chi)\left(\frac{V}{N}\right)^{\chi} N^{\sigma} + 2c_h(1+\chi_h) V_t^{\chi_h} N_t^{\sigma_h - \chi_h} \left(\frac{g(1-\phi_j)}{2} \right)^{1+\chi_h} \left(1 + \phi_j \varepsilon_{g,\theta} \right)}{g(\theta)(1-\phi_j)}$$

Plugging λ_t back into the FOC for wages yields.

$$w_j = (\varepsilon_{R,w} - \varepsilon_{S,w}) \left(c(1+\chi)\left(\lambda_{EE}\theta\right)^{1+\chi} N^{\sigma} + c_h(1+\chi_h) N^{\sigma_h} \left(\frac{\lambda_{EE} f(\theta)(1-\phi_j)}{2} \right)^{1+\chi_h} (\frac{1}{2} + \phi_j \varepsilon_{g,\theta}) \right) \tag{63}$$

FOC on Employment N

$$\begin{aligned}
\mathcal{L}_{N_{t,t}} : MRPL_t - w_t + & \beta_f c(\chi - \sigma) V_{t+1}^{1+\chi} N_t^{\sigma - \chi - 1} - \lambda_t + \beta_f \mu_{t,t+1}(1 - S(w_t)) \\
& + \beta_f c_h(\chi_h - \sigma_h) V_{t+1}^{1+\chi_h} N_t^{\sigma_h - \chi_h - 1} \\
& \times \left(g(\theta_{t+1}(V_{j,t+1})) \sum_{k \neq j} \frac{\partial \phi_{k,t}^n}{\partial N_{t,t}} \frac{\exp(\gamma \mathcal{V}_{j,t+1})}{\exp(\gamma \mathcal{V}_{j,t+1}) + \exp(\gamma \mathcal{V}_{k,t+1})} \right)^{1+\chi_h} \\
& + \beta_f c_h(1+\chi_h) V_{t+1}^{1+\chi_h} N_t^{\sigma_h - \chi_h} (\sum_{k \neq j} \frac{\partial \phi_{k,t}^n}{\partial N_{t,t}})^{\chi_h}
\end{aligned}$$

$$\times \left(g(\theta_{t+1}(V_{j,t+1})) \frac{\exp(\gamma \mathcal{V}_{j,t+1})}{\exp(\gamma \mathcal{V}_{j,t+1}) + \exp((\gamma \mathcal{V}_{k,t+1})} \right)^{1+\chi_h}$$
$$- \beta_f \lambda_{t+1} V_{t+1} g(\theta_{t+1}(V_{j,t+1})) \frac{\exp(\gamma \mathcal{V}_{j,t+1})}{\exp(\gamma \mathcal{V}_{j,t+1}) + \exp(\gamma \mathcal{V}_{k,t+1})} = 0$$

Under symmetry $\mathcal{V}_j = \bar{\mathcal{V}}$, $\lambda = \mu$, the above expression simplifies to

$$MRPL - w + c(\chi - \sigma) V^{1+\chi} N^{\sigma-\chi-1} + c_h V^{1+\chi_h} N^{\sigma_h - \chi_h - 1} \left(\frac{g(\theta)(1-\phi_j)}{2} \right)^{1+\chi_h}$$
$$\times \left((\chi_h - \sigma_h) + N(1+\chi_h)(1-\phi_j)^{-1} \right) - \lambda S - \lambda V g(\theta) \frac{1}{2} = 0$$

Taking advantage of the fact that $N_j = \phi_j$ and $R = \frac{g(\theta)}{2}(1-\phi_j)$,

$$MRPL - w + c(\chi - \sigma) V^{1+\chi} N^{\sigma-\chi-1} + c_h S^{1+\chi_h} N^{\sigma_h} \left((\chi_h - \sigma_h) + (1+\chi_h) \frac{\phi_j}{1-\phi_j} \right) - \lambda \frac{S}{1-\phi_j} = 0$$

Combining equations From our FOCs on w, V, and N, and recalling in an equilibrium with symmetric wages that $S = \frac{\lambda_{EE} f(\theta)}{2}(1-\phi_j)$ and $R = \frac{g(\theta)}{2}(1-\phi_j)$, this yields

$$\frac{w + c\left(\frac{V}{N}\right)^{1+\chi} N^{\sigma} + c_h \left(\frac{H}{N}\right)^{1+\chi_h} N^{\sigma_h}}{MRPL} =$$
$$\frac{\frac{\mathcal{C}_v}{N}\left(1 + (\varepsilon_R - \varepsilon_S)(1+\chi)\right) + \frac{\mathcal{C}_h}{N}\left(1 - \varepsilon_S(1+\chi_h)(1+2\phi_j \varepsilon_{g,\theta})\right)}{\frac{\mathcal{C}_v}{N}\left(1 + (\varepsilon_R - \varepsilon_S)(1+\chi) + \sigma + (1+\chi)\frac{\phi_j}{1-\phi_j}\right) + \frac{\mathcal{C}_h}{N}\left(1 - \varepsilon_S(1+\chi_h)(1+2\phi_j\varepsilon_{g,\theta}) + \sigma_h + \varepsilon_{g,\theta}(1+\chi_h)\frac{\phi_j}{1-\phi_j}\right)},$$

where $\mathcal{C}_{\mathcal{V}} \equiv c\left(\frac{V}{N}\right)^{1+\chi} N^{\sigma}$ and $\mathcal{C}_{\mathcal{H}} \equiv c_h \left(\frac{H}{N}\right)^{1+\chi_h} N^{\sigma_h}$ Without hiring costs ($c_H = 0 \to \mathcal{C}_H/N = 0$), this nests our equation for the recruiting cost-adjusted markdown is Section 4

$$\frac{w + c\left(\frac{V}{N}\right)^{1+\chi} N^{\sigma}}{MRPL} = \frac{1 + (\varepsilon_R - \varepsilon_S)(1+\chi)}{1 + (\varepsilon_R - \varepsilon_S)(1+\chi) + \sigma + (1+\chi)\frac{\phi_j}{1-\phi_j}}$$

With only atomistic firms ($\phi_j = 0$) but positive hiring costs, the recruiting cost-adjusted markdown becomes

$$\frac{w + c\left(\frac{V}{N}\right)^{1+\chi} N^{\sigma} + c_h \left(\frac{H}{N}\right)^{1+\chi_h} N^{\sigma_h}}{MRPL} = \frac{\frac{\mathcal{C}_v}{N}\left(1 + (\varepsilon_R - \varepsilon_S)(1+\chi)\right) + \frac{\mathcal{C}_h}{N}\left(1 - \varepsilon_S(1+\chi_h)\right)}{\frac{\mathcal{C}_v}{N}\left(1 + (\varepsilon_R - \varepsilon_S)(1+\chi) + \sigma\right) + \frac{\mathcal{C}_h}{N}\left(1 - \varepsilon_S(1+\chi_h) + \sigma_h\right)}.$$

Log Linearization with Hiring Costs When firms face both recruiting and hiring costs, the out of steady-state wage equation is

$$w_t = N_{t-1}\varepsilon_{R,w_t}\frac{S(w_t)}{N_{t,t}}c(1+\chi)\left(\frac{V_t}{N_{t-1}}\right)^{\chi+1}N_{t-1}^{\sigma} - \varepsilon_{S,w_t}S(w_t)^2\times$$
$$\sum_{\tau=1}^{\infty}\frac{c(1+\chi)\left(\frac{V_{t+\tau,t+\tau}}{N_{t+\tau-1}}\right)^{\chi}N_{t+\tau-1}^{\sigma} + c_h(1+\chi_h)\left(\frac{V_{t+\tau,t+\tau}}{N_{t+\tau-1}}\right)^{\chi_h}N_{t+\tau-1}^{\chi_h\sigma_h}R(w_{t+\tau})^{\chi_h+1}}{R(w_{t+\tau})}(1-S(w_t))^{\tau-1}$$

Solving for $\hat{w}_t$ gives:

$$\begin{aligned}\hat{w}_t =& \frac{1}{\Omega}\Bigg[\frac{(1+\chi)\frac{\mathcal{C}_v}{N}}{(1+\chi)(\varepsilon_{R,w}-\varepsilon_{S,w})\frac{\mathcal{C}_v}{N} - \varepsilon_{S,w}\frac{\mathcal{C}_h}{N}}\varepsilon_{R,w}(\chi\hat{h}_t + \sigma\hat{N}_{t-1}) + \varepsilon_{S,w}S\sum_{\kappa=0}^{\infty}(1-S)^{\kappa} \\ &\times\Bigg(\frac{(1+\chi)\frac{\mathcal{C}_v}{N}}{(1+\chi)(\varepsilon_{R,w}-\varepsilon_{S,w})\frac{\mathcal{C}_v}{N} - \varepsilon_{S,w}\frac{\mathcal{C}_h}{N}}\Big[(1+\chi)\varepsilon_{R,w}\hat{w}_{t+\kappa+1} - \chi\hat{h}_{t+\kappa+1} - \sigma\hat{N}_{t+\kappa}\Big] \\ &- \frac{(1+\chi)\frac{\mathcal{C}_h}{N}}{(1+\chi)(\varepsilon_{R,w}-\varepsilon_{S,w})\frac{\mathcal{C}_v}{N} - \varepsilon_{S,w}\frac{\mathcal{C}_h}{N}}\Big[\chi_h\hat{h}_{t+\kappa+1} + \sigma_h\hat{N}_{t+\kappa}\Big]\Bigg)\Bigg]\end{aligned}$$

where $\Omega = 1 - \frac{(1+\chi)\frac{\mathcal{C}_v}{N}}{(1+\chi)(\varepsilon_{R,w}-\varepsilon_{S,w})\frac{\mathcal{C}_v}{N} - \varepsilon_{S,w}\frac{\mathcal{C}_h}{N}}\varepsilon_{R,w}(\xi_R + \varepsilon_{S,w} - (1+\chi)\varepsilon_{R,w}) + \frac{(1+\chi)\frac{\mathcal{C}_v+\mathcal{C}_h}{N}}{(1+\chi)(\varepsilon_{R,w}-\varepsilon_{S,w})\frac{\mathcal{C}_v}{N} - \varepsilon_{S,w}\frac{\mathcal{C}_h}{N}}(\xi_S + \varepsilon_{S,w})\varepsilon_{S,w}$, so current wages are function of current and future hiring rates, recent, current, and future employment level, and the optimal wage of future cohorts.

C.4 Amenities

In this section, we sketch a model with on-the-job search, costly amenities, but no recruiting margin. We argue that the inclusion of amenities can create an infinitely elastic labor supply curve only in the special case where there are no exogenous separations ($s_0 = 0$). This is in contrast with the model with a recruiting margin but no amenities in the main text. The key difference is that amenities do not help firms overcome matching frictions.

We modify the model in the main text so that (i) firms post constant wages for each chort by assumption and (ii) workers now get utility from the level of amenities a_j when employed at firm j, with relative wage ξ. The value of a worker employed at firm j is

$$\mathcal{V}_j = \log w_j + \xi \log a_j + \beta_w \left(f(\theta')\lambda_{EE} \sum_{k \neq j}^{\mathcal{K}} \phi_k^{v'} \gamma^{-1} \phi_k^{v'} \log \left(\exp\left(\gamma \mathcal{V}_j'\right) + \exp\left(\gamma \mathcal{V}_k'\right) \right) + (1 - f(\theta')\lambda_{EE})\mathcal{V}_j' \right),$$

where $\phi_k^{v'}$ now indicates the shares of vacancies associated with a $\{w, a\}$ bundle.

The firm's problem is (suppressing the j):

$$\max_{\{\{N_{s,t}\}\},\{w_t\},\{a_t\}} \sum_{t=0}^{\infty} \beta^{\tau} \left(A \left(\sum_{s=-\infty}^{t} N_{s,t} \right)^{\alpha} - \sum_{s=-\infty}^{t} w_s N_{s,t} - c_a \frac{\sum_{s=-\infty}^{t} a_s N_{s,t}}{\left(\sum_{s=-\infty}^{t} N_{s,t}\right)^{\psi}} \right)$$

subject to

$$\begin{aligned} N_{s,t} &= (1 - S(w_s, a_s))N_{s,t-1} \\ N_{t,t} &= R(w_s, a_s)V_t. \end{aligned}$$

Here, $\psi \in [0, 1)$ allows for returns to scale in providing amenities: as the firm gets bigger, it may become cheaper to provide the same level of benefits per worker.

While a firm can increase the probability that workers accept a job and decrease endogenous separations with higher wages and better amenities, a greater level of amenities does not increase the number of matches, as search is random. Since random search limits worker inflows, any level of exogenous separations puts a hard limit on employment. Define V_0 as the exogenous amount of recruiting activity available to the firm. As in the main text, in steady state, inflows must equal outflows: $SN = VR$. As the firms offer better wages and amenities, $R \to 1$ and $S \to s_0$. The maximum employment of any firm is therefore $N^{max} = V_0/s_0$, and the marginal cost of employment becomes infinite because higher compensation (in the form of either wages or amenities) yields no additional workers.